Workshop on Arithmetic and Computation 2017

This small-scale workshop, which is jointly organised by the KU Leuven and the University of Luxembourg, brings together experts in computational aspects of Sato-Tate type questions and Hilbert modular forms.

Dates

15 - 16 June 2017

Place

KU Leuven, Departement Wiskunde, Celestijnenlaan 200B, 3001 Heverlee, Belgium
Rooms: B01.14 (lectures), B02.18 (lunch breaks)

Organisers

Jan Tuitman, Wim Veys, Gabor Wiese

Speakers

Samuele Anni (Heidelberg), Peter Bruin (Leiden), Lassina Dembélé (Bonn), Francesc Fité (Barcelona), Andrew Sutherland (MIT), Jasper Van Hirtum (Leuven/Luxembourg)

Programme

Thursday, 15 June 2017
10:00 - 10:30: Gabor Wiese *On the arithmetic of modular forms*
10:45 - 11:30: Jasper Van Hirtum *Computational Aspects of Classical and Hilbert Modular Forms*
11:30 - 12:30: Continuation of the PhD defence of Jasper Van Hirtum
15:00 - 16:00: Andrew Sutherland *Sato-Tate in dimension 3*
16:30 - 17:30: Peter Bruin *Towards a database of Galois representations on finite groups*
Conference dinner

Friday, 16 June 2017
10:00 - 11:00: Lassina Dembélé *On the compatibility between base change and Hecke action*
11:30 - 12:30: Francesc Fité *On the convergence rate towards the Sato-Tate measure*
14:00 - 15:00: Samuele Anni *Graphs of congruences*

Abstracts

Samuele Anni *Graphs of congruences*
The theory of congruences of modular forms is a central topic in contemporary number theory, lying at the basis of the proof of Mazur's theorem on torsion in elliptic curves, Fermat's Last Theorem, and Sato-Tate, amongst others. Congruences are a display of the interplay between geometry and arithmetic. In order to study them, in a joint work with Vandita Patel (University of Warwick), we are constructing graphs encoding congruence relations between newforms. These graphs have extremely interesting features: they help our understanding of the structure of Hecke algebras, and they are also a new tool in the study of numerous conjectures.
In this talk I will describe these new objects, show examples and explain the possible applications.

Peter Bruin *Towards a database of Galois representations on finite groups*
The goal of this talk is to explain a new way to explicitly describe representations of the Galois group of a number field K on finite Abelian groups. The objects in question are dual pairs of finite K-algebras; these are in principle equivalent to finite commutative group schemes or Hopf algebras, but are easier to compute and to store. This will make it feasible to build a database of such representations, which may eventually be included in the L-Functions and Modular Forms Database.

Lassina Dembélé *On the compatibility between base change and Hecke action*
Let F/E be a Galois extension of totally real number fields. In this talk, we will discuss the action of Gal(F/E) on Hecke orbits of automorphic forms on GL_2. This reveals some compatibility between base change and Hecke action, which has several implications for Langlands functoriality.

Francesc Fité *On the convergence rate towards the Sato-Tate measure*
Attached to an abelian variety A defined over a number field there is a compact real Lie subgroup G of the unitary symplectic group, the so called Sato-Tate group of A, whose Haar measure \mu conjecturally governs the asymptotic distribution of the Frobenius elements attached to A.
In this talk, under the assumption of certain standard conjectures on L-functions attached to the irreducible representations of G, we study the asymptotic convergence of any virtual character of G to the value predicted by \mu. We find that the convergence rate is dictated by several arithmetic invariants of A, such as its rank or the Sato-Tate group itself. The techniques that we use were introduced by Sarnak, in order to explain the Chebyshev bias in the sign of the Frobenius traces of an elliptic curve without complex multiplication defined over Q. We show that the same methods can be adapted to study the convergence rate of any virtual character of the Sato-Tate group of any abelian variety defined over a number field.
This is a joint work with Xevi Guitart.

Andrew Sutherland *Sato-Tate in dimension 3*
The Sato-Tate group of an abelian variety over a number field is a compact Lie group whose Haar measure is conjectured to govern the distribution of its Frobenius classes, and in particular, the limiting distribution of the normalized Euler factors that appear in its L-function. This conjecture has been proved in full generality only for abelian varieties of dimension 1 over totally real fields, but there has been substantial progress in dimension 2, including a complete classification of the 52 Sato-Tate groups that arise and proofs of equidistribution for at least one example of all but the generic case. I will report on recent results and work in progress in dimension 3, including some remarkable new (and very recent) computations.

Jasper Van Hirtum *Computational Aspects of Classical and Hilbert Modular Forms*
In this talk we present two results concerning computational aspects of modular forms.
In the first part, we present a heuristic model that settles the following question related to the Sato-Tate and Lang-Trotter conjectures: given a normalised classical eigenform of weight 2 with quadratic coefficient field, what is the asymptotic behaviour of the number of primes p such that the p-th coefficient of this eigenform is a rational integer? We provide an explicit model that describes the asymptotic behaviour in terms of the associated Galois representation and show that this model holds under reasonable assumptions.
In the second part, we discuss how one uses the underlying graded algebra structure of Hilbert modular forms to compute the adelic q-expansion of Hilbert modular forms of weight 1 as the quotient of modular forms of higher weights. This approach enables us to compute adelic q-expansions with complex coefficients for both parallel and partial weight, filing the gap left by standard computational methods. Moreover, we show that, for parallel weight, the results extend to adelic q-expansions with coefficients in finite fields. In particular, we prove that this can be done in all characteristics simultaneously. Finally, we provide explicit examples of non-liftable Hilbert modular forms of weight 1.

Gabor Wiese *On the arithmetic of modular forms*
In this short overview talk, we will stress the arithmetic significance of the coefficients of modular forms. This naturally leads to questions on the distribution of the coefficients in various senses. We will briefly touch on some of them and state some open questions.
The arithmetic information in the coefficients of a Hecke eigenform is summarised in the attached Galois representation. When studying its ramification properties, one notices that forms of weight one play a special role that we will explain.