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Author(s)*: Jérôme Dugast

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Keywords: news, order book, market depth, price variability, trading volume.

***Corresponding Author's Address:** Tel: +352 46 66 44 52 12 Fax : +352 46 66 44 6835
E-mail address: jerome.dugast@gmail.com

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For editorial correspondence, please contact: sophie.lux@uni.lu

University of Luxembourg
Faculty of Law, Economics and Finance
Luxembourg School of Finance
6 rue Coudenhove Kalergi
L-1359 Luxembourg

Unscheduled News and Market Dynamics*

Jérôme Dugast†

November 11, 2017

Abstract

When unscheduled news arrives, investors react with a stochastic delay yet still may exploit new information. In this context, I study the equilibrium dynamics of limit order markets. Idiosyncratic liquidity shocks occur continuously, and news arrives at random times. Competition to accommodate next-instant liquidity needs pushes the bid-ask spread down to the tick. Following news, order flows become imbalanced and market depth is consumed, resulting in positive covariance between price variability, trading volume, and order book imbalances. Holding the unconditional price variability constant, news frequency has a negative effect on both market depth and the variability-volume covariance.

KEYWORDS: news, order book, market depth, price variability, trading volume.

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†Luxembourg School of Finance, University of Luxembourg, Campus Kirchberg (Bloc F), 6 rue Coudenhove-Kalergi, L-1359 Luxembourg. Phone: (+352) 46 66 44 5212; E-mail: jerome.dugast@uni.lu.

1 Introduction

In modern financial markets, most stocks, as well as bonds and derivatives, are traded via electronic limit order books. In such a trading venue, any participant can provide new quotes, with limit orders, or trade against existing ones, with market orders. Limit orders give the possibility to improve the execution price but potentially expose the investor to a winner's curse problem when public information arrives. What underlies the placement of limit and market orders when information is released, at random times, by *unscheduled news* (e.g. on Bloomberg, Reuters Newswire,...)? How do news arrivals, and their frequency, impact bid-ask spread, market depth, price discovery, volatility and trading volume? This paper is the first to tackle these questions in the context of an equilibrium model, in which all agents are rational and can use fully dynamic strategies, involving market orders as well as the placement and cancellation of limit orders.

My model of a limit order market builds on the over-the-counter market framework of Duffie, Gârleanu, and Pedersen (2005, 2007), hereafter referred to also as "DGP". My model differs from DGP in that investors trade on a centralized limit order market, and that it introduces unscheduled news arrival, which is novel in the literature. In the spirit of DGP, all sources of uncertainty are modeled as Poisson processes, in particular the time at which news may arrive, and the delay in each investor's reaction following that news. These modeling choices offer tractability and allow me to describe the joint dynamics - around news arrival - of the order book, order flows, trading volume, and price.

Prior to the arrival of news, the market is in a steady state and trading takes place. Both market and limit orders are used in equilibrium. As in DGP, at every instant a constant fraction of the investors' pool incurs a shock in private valuation and thus needs to trade. From the steady-state condition it follows that a constant share of these investors trades with market orders and the remaining share places limit orders. As a result, the flows of buy and sell market orders are continuous and nonvarying. Under these circumstances, the equilibrium bid-ask spread must be equal to its minimal value: one tick. Indeed, if there were an available trading price between the best bid and ask prices, then an investor who places a market order in equilibrium would prefer to submit a limit order at this price - and that order would be immediately executed by the contemporaneous flow of market orders. In equilibrium, such a profitable deviation cannot exist. Nowadays, a one tick bid-ask spread is empirically realistic. For instance, O'Hara, Saar and Zhong (2015) document that stocks with a large tick size relative to its price exhibit a one tick bid ask spread more than 50% of the time.

In my model, investors face a trade-off between limit and market orders. Consider an investor who wants to buy. If he uses a buy limit order, upon execution he saves the one-tick bid-ask spread compared to a buy market order. However, the order incurs an execution delay that materializes in adverse selection risk if news arrive meanwhile. Indeed, the order is exposed to picking-off risk following negative news, and to non-execution risk following positive news, as in Foucault (1999). Yet, agents have the possibility to react to news. With some probability, the former investor can, in case of negative news, cancel his buy order and avoid being picked-off, or, in case of positive news, cancel his order, send a buy market order before price change, and avoid non-execution. Thus, the possibility to react to news mitigates the risks associated with limit orders. This reaction to news is also beneficial to market order users. Indeed, if the investor uses a buy market order, following negative news, with some probability he can resell his asset share and wait for the price to adjust down to buy again. Contrariwise, following positive news, the investor keeps the asset. Importantly, in case of negative news, whether the investor has used a buy limit order or a buy market order, outcomes are similar: ex-post, he has overpaid for the asset unless he has managed to react and has either canceled his limit order or undone his previous trade. Thus, these outcomes cancel out in the trade-off between limit and market buy orders. The trade-off is symmetric for sell orders. Overall, compared to a market order, a limit order suffers from a *relative* adverse selection cost that arises because of non-execution risk due to news arrival.

Limit orders queue before execution. In equilibrium, the limit order execution speed adjusts so that likely saving the one-tick bid-ask spread compensates for the limit order's relative adverse selection cost. If this cost increases, then the limit order execution speed must increase, so that investors remain indifferent between market and limit orders. As this execution speed increases, so does the flow of limit orders leaving the book, which reduces market depth.

When news hit the market, investors observe it only after some delay. This generates a transition phase, during which investors react to the news by canceling their own stale limit orders and picking-off those remaining in the book. Once all stale limit orders have been canceled or executed, the new equilibrium bid and ask prices are established, ending the transition phase. The greater the market depth prior to news arrival, the more stale limit orders must be canceled or executed and the longer the transition phase.

More frequent news arrivals imply a larger adverse selection cost for limit orders. This results in lower depth and also shortens the transition phase. During the transition phase, the flow of limit order cancellations is proportional to the market depth; hence this flow decreases as news arrives more frequently. At the same time, however, the flow of market orders does not depend on the news arrival frequency. So in the transition phase, the trades' share in the orders' outflow (i.e., trades + cancellations) increases when the news arrival frequency increases.

In equilibrium, when news hit the market, trading volume goes up, as stale limit orders get picked off. Thus, the model offers a micro-foundation for the time series correlation between volatility and volume. Investors, however, rationally anticipate the cost of stale limit orders. Hence, in the cross section, as the frequency of news increases, limit orders are deterred and trading volume is reduced.

Following the arrival of positive news, investors hit stale limit sell orders. Consequently, there is more activity (cancellations and executions) on the ask side than on the bid side of the book, and market depth is depleted on the ask side relative to the bid side. This continues until the end of the transition phase, at which point, new and higher ask and bid quotes are established. Symmetric effects take place after negative news. Thus, empirically, a flurry of activity on the ask (resp. bid) side of the book predicts an increase (resp. decrease) in prices.

My paper contributes to theoretical literature on dynamic limit order markets and is the first (to my knowledge) that studies the effect of unscheduled news. This model has the advantage of tractability without being too restrictive on the action set of agents. The model is tractable because (i) I use a setup à la DGP in which order flows are continuous and (ii) I focus on equilibria in which aggregate preferences are in a steady state, since that reduces the dimensionality of the state space. Biais, Hombert, and Weill (2014) offer a dynamic limit order market model that also builds on DGP. Their paper differs from mine in that they study the equilibrium dynamics generated by a shock on aggregate preferences.

Parlour (1998) and Foucault (1999) were the first to posit models of limit order markets designed as dynamic games. These models make several restrictive assumptions to improve tractability; for instance, they do not allow for limit order cancellation. Foucault, Kadan, and Kandel (2005) focus on the dynamics of liquidity supply in a limit order market. Investors have different preferences for immediacy, which determines their order choice. However, news arrival does not play a role in their model. Rosu (2009) designs a continuous-time model in which traders can freely send limit orders at any price and can cancel them at any time. He obtains greater tractability by increasing the investors' action set. Rosu (2015) augments his previous model with a varying common value of the asset. That approach allows him to address the effect of privately informed trading. In contrast, I study the effect of public information to which everyone has equal access but with some randomness. Goettler, Parlour, and Rajan (2005, 2009) computationally solve a limit order market model that accommodates heterogeneous investor preferences, order sizes, and information asymmetries. In Goettler et al., the dimensionality of the state space problem prevents from solving the model analytically.

Similarly to my paper and to Biais, Hombert, and Weill (2014), Pagnotta and Philippon (2015) model centralized markets following the approach of DGP. These models contribute to the stream of theoretical literature that extends the DGP framework to focus on over-the-counter (OTC) markets (Weill 2007; Vayanos and Weill 2008; Weill 2008; Lagos and Rocheteau 2009; Lagos, Rocheteau, and Weill 2011). This body of research may eventually offer a unified framework for studying the dynamics of financial markets more generally.

My paper also contributes to the literature on the reaction of financial markets to public information releases. It is noteworthy that most of this literature focuses on scheduled releases, such as macroeconomic or earnings announcements (Ederington and Lee 1995; Fleming and Remolona 1999; Green 2004; Della Vigna and Pollet 2009). A few papers consider unscheduled news. Gross-Klussmann and Hautsch (2011) study the intraday effects of unscheduled news. Consistently with my model, they observe spikes in trading volume and volatility around news events. Graham, Koski, and Loewenstein (2006) empirically compare the effects of anticipated and non-anticipated dividend announcements. In my model, if news were scheduled then investors who placed limit orders would optimally cancel them just before the announcement. In line with this result, Graham and al. find that market depth decreases before anticipated announcements and after non-anticipated announcements. The implication is that unscheduled and scheduled news must be addressed separately.

The rest of the paper is organized as follows. Section 2 presents the model, and Section 3 solves for the equilibrium. In Section 4 I develop the empirical implications, after which assumptions and possible extensions are discussed in Section 5. Section 6 concludes.

2 Model

This section presents the model's four building blocks: the asset and its payoff structure, investors' preferences, the delay in investors' reaction to news, and the trading mechanism. I also define investors' trading strategies.

2.1 The Asset Value Dynamics

I consider the market for a risky asset in a continuous time framework. The asset can be traded during the time interval, $[0, \tau_{end}]$, where the termination date, τ_{end}, is random and follows a Poisson process of intensity, r. At date τ_{end}, a share of the asset pays off its current value[1], $v_{\tau_{end}}$. The *asset value*, v_t, evolves over the time interval because of news arrival.

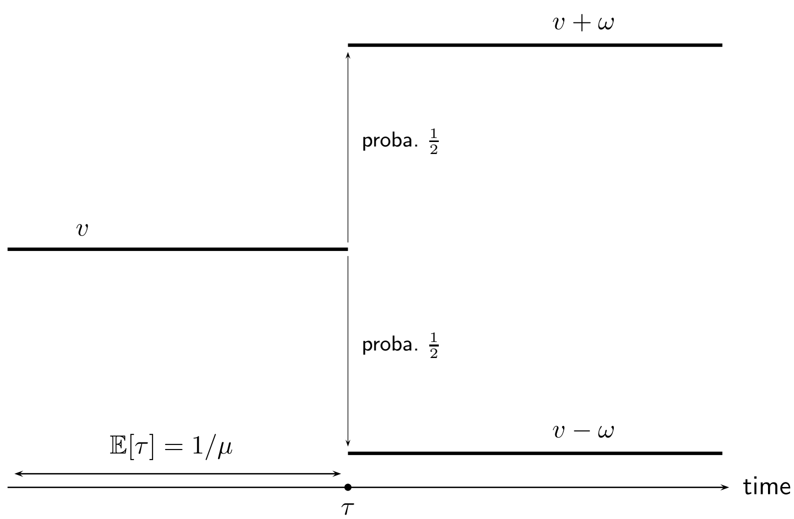

Figure 1: The Asset value dynamics.

I model news by the asset value's single-jump process dynamics. At the beginning of the game ($t = 0$), the asset value is equal to v; that value does not change until the arrival of news, which occurs at time τ. Time τ is random and follows a Poisson distribution of intensity μ, where μ is the news arrival frequency. The asset value jumps (up or down) at

[1]This initial setup is equivalent to Duffie, Gârleanu, and Pedersen (2005); in that paper, the horizon is infinite, investors are infinitely lived and have a time discount rate r, and the asset value v is a utility flow for the investor who holds one share. Returning to my setup requires only that one replace v with v/r.

time τ, and the new asset value is $v + \omega$ or $v - \omega$ with equal probability; here ω is the *jump size*. Following news arrival, the asset value remains unchanged for any $t \geq \tau$. Figure 1 illustrates the asset value dynamics.

In my model, news arrival actually occurs if and only if it takes place before the game ends, that is if $\tau < \tau_{end}$. Hence the probability of news arrival (i.e., the likelihood that news arrives before the game ends) is

$$\mathbb{E}[\mathbb{I}_{\tau<\tau_{end}}] = \frac{\mu}{r+\mu}. \quad (1)$$

2.2 Investors' preferences

There is a unit mass of risk-neutral investors who can trade the asset. As in Duffie, Gârleanu, and Pedersen (2005, 2007), an investor is characterized by her private valuation of the asset, which can be high or low. An investor with a low private valuation continuously incurs a holding cost δ per asset unit; an investor with a high private valuation incurs no such cost. At each instant, an investor's private valuation may switch from high to low - or from low to high - at a Poisson intensity ρ.

The case on which I focus is that where the initial distribution of preferences is the stationary one where half of the population has a high private valuation of the asset while the other half has a low private valuation. In aggregate, then, the distribution of preferences remains unchanged throughout the game.

Asset supply and holding constraint. I follow DGP in assuming that an investor can own either one unit or zero units of the asset. I assume also that the asset supply is equal to 1/2. Therefore, half of the population owns the asset at any particular time.[2]

Investors' types. In the rest of this paper, an investor's *type* is defined as the combination of his asset-holding status and his private valuation of the asset. Each investor's type belongs to the set $\{ho, hn, lo, ln\}$, where h and l denote "high" and "low" and where o and n signify "owner" and "non-owner". Hence the mass of investors can be divided into four populations: L_{ho}, L_{hn}, L_{lo}, and L_{ln}. Because there is a unit mass of investors, these four populations must

[2]Setting the asset supply to 1/2 renders the problem symmetric between buyers and sellers of the asset. By a continuity argument, the results reported here should hold for any supply $s \in [0, 1]$ with s "close enough" to 1/2.

sum to unity:

$$L_{ho} + L_{hn} + L_{lo} + L_{ln} = 1. \tag{2}$$

Since the asset supply is 1/2, it follows that the mass of asset owners is - at any moment - half of the unit mass of investors:

$$L_{ho} + L_{lo} = \frac{1}{2}. \tag{3}$$

Finally, the mass of high-private valuation investors is, at any point in time, equal to 1/2,

$$L_{ho} + L_{hn} = \frac{1}{2}. \tag{4}$$

Overall, there are three independent equations, (2), (3), and (4), and four unknowns, L_{ho}, L_{hn}, L_{lo} and L_{ln}. To determine the four populations at equilibrium, there remains one free parameter to pin down. Proposition 1 derives this free parameter, which is denoted α_t.

Proposition 1. *At each instant t, there is an $\alpha_t \in \mathbb{R}$ such that the different masses of population satisfy*

$$L_{hn} = L_{lo} = \alpha_t, \text{ and } L_{ho} = L_{ln} = \frac{1}{2} - \alpha_t. \tag{5}$$

α_t must satisfy the constraints of non-negativity for the populations,

$$\frac{1}{2} - \alpha_t \geq 0, \text{ and } \alpha_t \geq 0. \tag{6}$$

At this stage, the trading mechanism has not been specified. The equilibrium value of α_t results from additional building blocks.

2.3 Market contacts and reaction delays

Whenever an investor's private valuation changes, she *contacts* the market - that is, she checks the state of the order book, observes the asset value, and decides whether or not to take action. In addition, an investor reacts to news: independently of a change in private valuation, she contacts the market (once) following news arrival. However, that reaction is not immediate. The delay in reaction may be due to technological frictions that prevent news from instantaneously reaching the investor's server and/or the investor's order from instantaneously reaching the exchange. The following assumption precisely defines contacting-time dynamics.

Assumption 1. *An investor contacts the market in either of the following cases:*

(i) when the investor' private valuation changes, at Poisson intensity ρ, or

(ii) after the investor becomes aware of news arrival at time $\tau + \Delta t$. Here Δt is a random delay, specific to each investor, that follows a Poisson distribution of intensity λ.

An investor who contacts the market will observe the state of the market and the asset value, and he can take an action.

Assumption 1 may seem restrictive and deserves therefore some explanations. If, before news arrival and after the reaction to news - that is, for $t < \tau$ and $t > \tau + \Delta t$ - investors could contact the market more often than only for private valuation changes (for instance continuously) then the equilibrium outcome would be the same. In the absence of news, investors find it optimal to trade only when their private valuation changes.

Following news arrival, it is possible for an investor to contact the market between τ and $\tau + \Delta t$ and thus to become aware of the news arrival before $\tau + \Delta t$. That would be the case if the investor's private valuation changed after news arrival but before the "λ contact". This assumption is made for tractability. Hereinafter, I consider the case where the reaction delay is small, i.e, $\lambda \gg \rho$, which makes these events negligible.

In practice, the reaction delay λ may be interpreted as the trading infrastructure's latency - which these days is measured in milliseconds (Hendershott and Moulton 2011; Riordan and Storkenmaier 2012). It is therefore reasonable to consider the case where λ is large compared to other Poisson intensities (e.g., ρ and μ).

2.4 The Limit Order Book

Trading takes place via a limit order book. All traders' quotes must be posted on a grid. The minimum difference between two prices is the tick size, Δ. Investors can use limit or market orders to trade. A limit order specifies a limit price at which the order can be executed, and that order remains posted in the order book until it is matched with a market order or is cancelled. Since each order is for a single share, the *depth* of the limit order book at price P, denoted D_P, is the number of limit orders posted at price P. Market orders do not specify a price limit, so they are executed immediately at the best quote on the opposite side.

Assumption 2. *The price grid is the set $\mathcal{P} = \{n\Delta,\ n = 0, 1, 2, \ldots\}$. Let A and B be defined as*

$$B = v - \frac{\delta}{2r} - \frac{\Delta}{2}, \quad A = v - \frac{\delta}{2r} + \frac{\Delta}{2}, \tag{7}$$

and $A^u = A + \omega$, $B^u = B + \omega$, $A^d = A - \omega$, $B^d = B - \omega$. I assume that A, B, A^u, B^u, A^d and B^d belong to $\mathcal{P}$.

Under Assumption 2, the average private valuation of the asset[3] - which is equal to $v - \delta/2r$ - is equidistant from B and A (i.e., the equilibrium bid and ask prices). Hence this assumption makes it easier to solve for the equilibrium.

Assumption 3. *Limit orders are executed according to a "random matching" rule. In particular: when a market order hits the book, all limit orders submitted at the same price have the same probability of being executed - that is, regardless of their respective submission dates.*

Assumption 3 simplifies the analysis because two investors of the same type and with a limit order in the book at the same price will have the same value function.

In practice, of course, time priority does apply. In Section 5, I discuss how the model can be solved with time priority and show that the main economic mechanisms remain unchanged. That being said, time priority does not apply across trading platforms in fragmented markets. Thus the real-world matching of the aggregate flow of market and limit orders may reflect a mix of time priority and random matching.

Assumption 4. *The holding cost, δ, and the tick size, Δ, are such that*

$$\delta > (r + 2\rho)\Delta \tag{8}$$

Assumption 4 states that δ is large when compared with Δ. This assumption ensures that the gain from trade, which has an order of magnitude of δ, is large enough compared to the bid-ask spread, which in equilibrium is equal to the tick size Δ. Investors prefer not to trade when those relative sizes are reversed.

Assumption 5. *The jump size, ω, is such that,*

$$\omega > \hat{\omega}(\delta, \Delta), \tag{9}$$

[3]Given the symmetry of the population's distribution, the average investor - the one with the average preference - would constantly incur a holding cost $\delta/2$ per asset unit. This hypothetical investor values the asset $E[v_\tau] - \delta/2r$, also equal to $v - \delta/2r$, because of the symmetrical pay-off distribution.

where $\hat{\omega}(\delta; \Delta)$ is a function determined in the proof of Proposition 8.

Assumption 5 ensures that ω is large enough so that, following news arrival, investors trade on news and possibly even though they would trade in the opposite direction if they were following their private valuation.

2.5 Investors' Trading Strategies

I now define an investor's action set and strategy as well as the equilibrium concept.

Action set. Whenever an investor contacts the market, she can take actions. These actions can be any combination of *elementary actions*, though they are performed under two constraints.

- As an owner, the investor can take only the following elementary actions: do nothing and remain an owner; submit a sell limit order; submit a sell market order and become a non-owner, if there is a buy limit order in the order book; or cancel her previous sell limit order.

- As a non-owner, the investor can take only the following elementary actions: do nothing and remain a non-owner; submit a buy limit order; submit a buy market order and become an owner, if there is a sell limit order in the order book; or cancel her previous buy limit order.

- The constraints on the action set are that the investor (i) own either zero or one asset share and (ii) have at most one limit order in the order book.

An investor can, in theory, play any sequence of elementary actions. However, in equilibrium and under constraints (i) and (ii), strategies will involve short and intuitive sequences of elementary actions.

Strategy. An investor's strategy consists of a mapping to the action set from the investor's history and current type, the current state of the market, and the clock time. A formal definition is provided in the Online Appendix (Section A).

Equilibrium concept. The focus in this paper is on *Markov perfect equilibria*,[4] in which an investor's strategy depends only on these three state variables: the investor's current type; the current asset value v_t; and the current aggregate state of the order book. I do not consider strategies that respond to unilateral deviations off the equilibrium path.[5]

3 Equilibrium

I begin this section by providing the reader with an overview of the model's focal equilibrium. Then, more details are given about the equilibrium's different phases, starting from the last phase (since the model is solved backwards). In Section 3.3 I describe the equilibrium steady state toward which the market converges following the arrival of news -after which I show, in Section 3.4, how the market converges toward that steady state once prices have adjusted to the new information. Section 3.5 focuses on the transition dynamics of the market during the phase between news arrival and the price adjustment to news. Finally, in Section 3.6 I describe the initial steady-state phase.

3.1 Equilibrium overview

This section briefly presents the three main phases of the dynamic equilibrium: (i) the initial steady state phase, (ii) the transition phase following news arrival, and (iii) the convergence towards a new steady state. Figure 2 depicts the equilibrium's different phases.

I solve the model by guessing (and then verifying) that,

i. Before news arrival (i.e., for $0 \leq t < \tau$), the market is in a steady state. B and A (see equation (7)) are the only bid and ask prices at which limit orders are submitted. Investors trades because of gains from trade due to differences in private valuation. They are indifferent between limit and market orders. The steady state is characterized by the equilibrium limit order execution rate, l_{eq}, and the market depth, α_{eq}; note that the latter term is also the equilibrium value of the free parameter defined in Proposition 1. The limit order execution rate, which determines the average execution delay of a limit order, must be such that investors are indifferent between limit and

[4]In the Online Appendix (Section A), I verify that the "one-shot deviation principle" applies: a strategy admits no profitable deviation if and only if it admits no profitable deviation that lasts for "one period".

[5]For instance, I do not consider any equilibrium that can be sustained only through a credible threat of punishment.

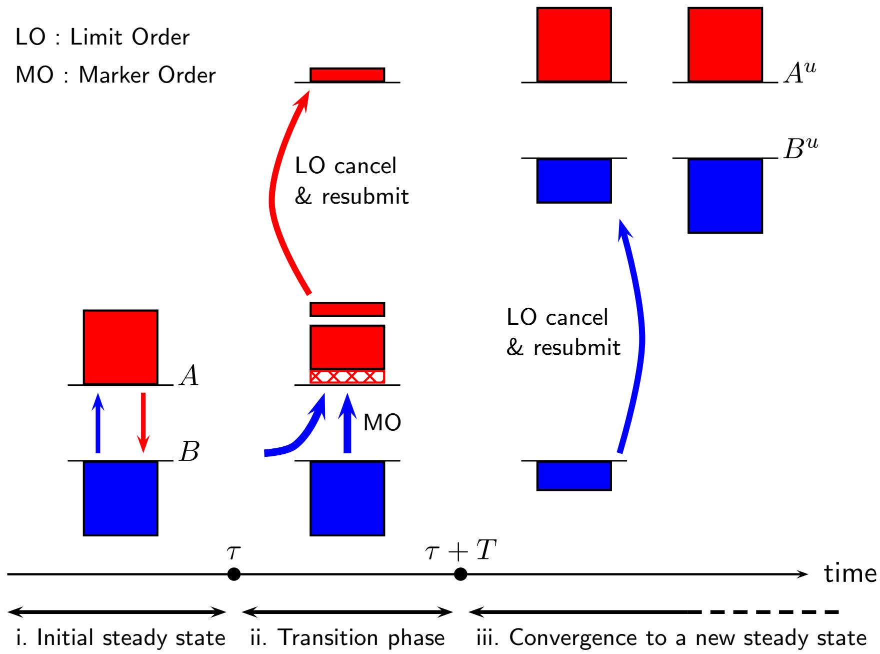

Figure 2: Equilibrium dynamics around positive news arrival

market orders. Because that execution rate depends on market depth, the terms l_{eq} and α_{eq} are jointly determined.

ii. Following news arrival, the market starts a transition phase that precedes the price adjustment to news and lasts for a period T. For $\tau \leq t < \tau + T$, if the new asset value is $v+\omega$ (resp., $v-\omega$) then sell (resp., buy) limit orders posted at price A (resp., B) offer a profit opportunity. It follows that eventually they are either executed or canceled and then resubmitted at a higher price $A^u = A + \omega$ (resp., a lower price $B^d = B - \omega$). The duration T of the transition phase is endogenously determined and corresponds to the time it takes for those stale orders to disappear from the order book.

iii. After the transition phase (i.e., for $t \geq \tau + T$), the bid and ask prices are B^u and A^u (or B^d and A^d, resp.), in line with the new asset value. During this phase, investors trade again because of differences in private valuations. And, in the absence of news, market depth converges to its steady state level.

Given its importance to the equilibrium, the interdependence of α_{eq} and l_{eq} deserves further comment. First, the limit order execution rate has a positive effect on the flow of orders leaving the order book and thus a negative effect on market depth. Second, market depth also corresponds to the initial number of stale orders at the beginning of the transition phase, which means that market depth determines the duration T of the transition phase. The shorter that phase, the riskier it is to place a limit order. Therefore, maintaining the indifference condition requires that such an order be compensated by a larger l_{eq}.

3.2 Steady state and one-tick market

Here I begin by defining the notion of market in steady state. I then show that, in a steady-state phase, the bid-ask spread must be equal to one tick in equilibrium.

Definition 1. *The market is in steady state when the aggregate state of the order book (trading prices, market depths) and the aggregate order flow (submissions, executions, cancellations) are deterministic and constant over time.*

During a steady state phase, the best bid and ask prices, B and A, do not change and market depths at these prices, D_A and D_B, remain constant. Even though the order book looks always the same in aggregate, trading occurs in equilibrium. Moreover, the order flow is constant. For instance, at each instant t, the same flows (i.e. quantities per infinitesimal time interval dt) M_A of buy market orders and M_B of sell market orders are submitted. The following proposition shows that, when trading occurs on both sides of the market - that is, $M_A > 0$ and $M_B > 0$ - the bid-ask spread must be equal to one tick.

Proposition 2. *If the market is in steady state in an equilibrium in which trading occurs on both sides of the market at each instant (i.e., $M_A > 0$ and $M_B > 0$), then its bid-ask spread is equal to one tick, i.e., $A - B = \Delta$.*

Proof. Consider an investor who finds it optimal to send a buy market order at price A; the existence of such an investor is guaranteed because trading occurs on both sides of the market. If there is a price P on the grid such that $B < P < A$, then the investor would be strictly better-off sending a limit order at P. Indeed, he expects [6] his order to be immediately

[6] The fact that deviation does not trigger a punishment is implicitly used here. One could see that these equilibria would not be trembling-hand robust. If investors would undercut by mistake with a small probability, as there is continuum of investors, undercutting would occur almost certainly and the punishment would be triggered permanently. Hence, the incentive to not deviate would break. Another argument: if I add a set (of infinitesimal size) of investors, who participate to the market only once during the game, they do not care about a potential punishment and immediately undercut.

hit by the flow of sell market orders, M_B. With this strategy, the investor buys at a lower price, holds the same quantity of the asset, and saves $A - P > 0$. News does not affect this outcome because the probability is 0 that news arrives at the same instant - and that result contradicts the optimality of the equilibrium strategy, proving the proposition. □

In traditional micro-structure models à la Glosten and Milgrom (1985) or Kyle (1985), adverse selection increases the bid-ask spread. In contrast, even though my model features a one-tick bid-ask spread, market depth depends on adverse selection (due to news). The key difference between my model and traditional ones is the continuous - not sequential - flow of investors submitting market orders. Otherwise, the time interval between two market order arrivals would be strictly positive. In that case, even the most competitive limit order, not competing with other order at the same price, would wait to be executed and so would be exposed to news.

3.3 The Steady state without news

In this section I describe the equilibrium steady state without news arrival - that is, when $\mu = 0$. This state corresponds to the asymptotic steady state toward which the market converges following the price change triggered by news arrival. Without loss of generality, I consider the equilibrium bid and ask prices B and A (as defined in eq. (7)) instead of B^u and A^u or B^d and A^d. I use D_A and D_B to denote the market depth at prices A and B.

Steady state strategy. Investors can trade at price A or price B. At these prices, investors of type *hn* and *lo* want to trade. They play a mixed strategy between limit and market orders, and after trading they become investors of respective types *ho* and *ln*. Yet at these prices, investors of type *ho* or *ln* are better-off not trading. My next proposition specifies the investors' equilibrium strategies.

Proposition 3. *Upon a change in preferences, an investor contacts the market and takes one of the following actions:*

- *If her type is ho, then she cancels any sell limit order she has in the book and stays out of the market thereafter.*
- *If her type is ln, then she cancels any buy limit order she has in the book and stays out of the market thereafter.*

- *If her type is hn, then she places either a buy market order (at price A) with probability m_A or a buy limit order (at price B) with probability $1 - m_A$.*

- *If her type is lo, then she places either a sell market order (at price B) with probability m_B or a sell limit order (at price A) with probability $1 - m_B$.*

Investors of type ho or ln never have a limit order in the order book. Investors of types lo and hn either (a) use a one-unit market order and immediately become respective types ln and ho or (b) post a one-unit limit order in the book. In equilibrium, then, the market depth at prices A and B is equal to the number of investors with types lo and hn, respectively; that is, $D_A = L_{lo}$ and $D_B = L_{hn}$. Moreover, the aggregate state of the limit order book does not change over time; hence D_A and D_B are constant. Proposition 1 implies that there exists an $\alpha \in \mathbb{R}$ such that $L_{lo} = L_{hn} = \alpha$. Thus the free parameter α is determined as the equilibrium market depth.

At each instant, the flows of investors whose private valuation changes - and hence need to buy or sell - are (respectively) equal to ρL_{ln} and ρL_{ho}, that is $\rho\left(\frac{1}{2} - \alpha\right)$ in both cases. It then follows from the strategies employed by investors that the flows of buy and sell market orders are $m_A \rho L_{ln}$ and $m_B \rho L_{ho}$. According to Proposition 2, the equilibrium bid-ask spread must be equal to one tick.

Limit order execution rates. Given the investors' strategies and the market depth α, I can define the *execution rate* of a sell limit order at price A, denoted l_A, as the intensity of the Poisson process that determines the execution time. At each instant, the flow of types ln who become hn and contact the market is equal to $\rho\left(\frac{1}{2} - \alpha\right)$. A fraction m_A sends market orders. As the "random matching" rule applies, a given sell limit order has the same probability of execution as any other order within the mass α. The intensity of execution is therefore written as,

$$l_A = \frac{m_A \rho\left(\frac{1}{2} - \alpha\right)}{\alpha}. \tag{10}$$

The execution rate of a buy limit order, denoted l_B, is similarly defined as

$$l_B = \frac{m_B \rho\left(\frac{1}{2} - \alpha\right)}{\alpha}. \tag{11}$$

In the analysis to follow, I solve for the limit order execution rates l_A and l_B and not for

m_A and m_B. This approach is more convenient because l_A and l_B are immediately evident from investors' value functions, as described next, and facilitate derivation of the indifference condition. More precisely, limit orders offer a better execution price than market orders but incur an execution delay. Once the execution rates l_A and l_B and the depth α are determined in equilibrium, one can then calculate m_A and m_B using equations (10) and (11).

Value functions. Equilibrium strategies of the four investor types generate a system of Bellman equations that define the value functions for each type. These value functions depends on the limit order execution rates. The equilibrium values of l_A and l_B are,determined by the indifference conditions between limit and market orders for investors of type lo or hn.

At each instant, an investor of type ho may receive - with Poisson intensity r - the payoff v from the asset. He may (with intensity ρ) also become a lo type and thereby obtain the continuation value V_{lo}. Hence the investor's value function V_{ho} is such that

$$(r+\rho)V_{ho} = rv + \rho V_{lo}. \tag{12}$$

If an investor of type hn has placed a buy limit order at price B then, at each instant, her order may (with Poisson intensity l_B) be executed. In the event that it is executed, the investor becomes an ho type and her continuation value is then $V_{ho} - B$. Alternatively, with intensity ρ she may become an ln type; then her value function V_{hn} is such that

$$(r+\rho+l_B)V_{hn} = \rho V_{ln} + l_B(V_{ho} - B). \tag{13}$$

If a type-hn investor has instead placed a market order, then her continuation value is $V_{ho}-A$. In deciding which order to place, the investor compares the value of a limit order (as given by equation (13)) with the value of the market order net of the payment A. Thus she compares, conditional on all possible next events, the difference in continuation values between the market order net of the payment A and the limit order. Here the possible events are: (i) end of the game, with intensity r; (ii) change in private valuation, with intensity ρ; or (iii) limit order execution, with intensity l_B. All these conditional differences sum to the unconditional difference in continuation value, $V_{ho} - A - V_{hn}$, as follows:

$$(r+\rho+l_B)(V_{ho} - A - V_{hn}) = r(v-A) + \rho(V_{lo} - A - V_{ln}) + l_B[\overbrace{V_{ho} - A - (V_{ho} - B)}^{=-\Delta}]. \tag{14}$$

The investor is indifferent between limit and market orders if and only if $V_{hn} = V_{ho} - A$. Similarly, a type-lo investor is indifferent between limit and market orders if and only if $V_{lo} = V_{ln} + B$. Incorporating these conditions into equation (14) yields

$$(\rho + l_B)\Delta = r(v - A). \tag{15}$$

The right-hand side of equation (15), $r(v - A)$, is the benefit of a market order: conditional on the end of the game, it yields the net benefit of trade $(v - A)$ whereas a limit order does not. The left-hand side, $(\rho + l_B)\Delta$, is the benefit of a limit order: conditional on limit order execution, it saves the investor Δ as compared with a market order. Conditional on a private valuation change, the limit order is simply cancelled; with a market order, in contrast, the asset share must be sold again - and thus incurs the round-trip cost (i.e., the bid-ask spread Δ). In equilibrium, the limit order execution rate l_B adjusts to obtain the indifference condition as follows:

$$l_B = \frac{rv - rA - \rho\Delta}{\Delta} \tag{16}$$

In the same way, we can calculate the value functions of investor types lo and ln, where the equilibrium execution rate is now

$$l_A = \frac{rB - \rho\Delta - rv + \delta}{\Delta} \tag{17}$$

The following proposition stipulates expressions for the equilibrium execution rates when A and B are replaced by their respective expressions in (7). Equilibrium value functions are derived in the proof of the proposition (see the Appendix).

Proposition 4. *In the steady state without news, both equilibrium limit order execution rates have the same value*

$$l_B = l_A = l^* = \frac{\delta - (r + 2\rho)\Delta}{2\Delta} \tag{18}$$

One can see from equation (18) that, when the tick size increases, the equilibrium execution rate is lower. After all, a larger tick size implies a larger bid-ask spread - which makes limit orders more valuable than market orders. The limit order execution rate decreases so that both order types remain equally valuable. Parameter δ measures the difference in private valuations; the higher is δ, the larger the gain from trade and the costlier to wait before execution. A higher limit order execution rate is needed to compensate for a higher δ.

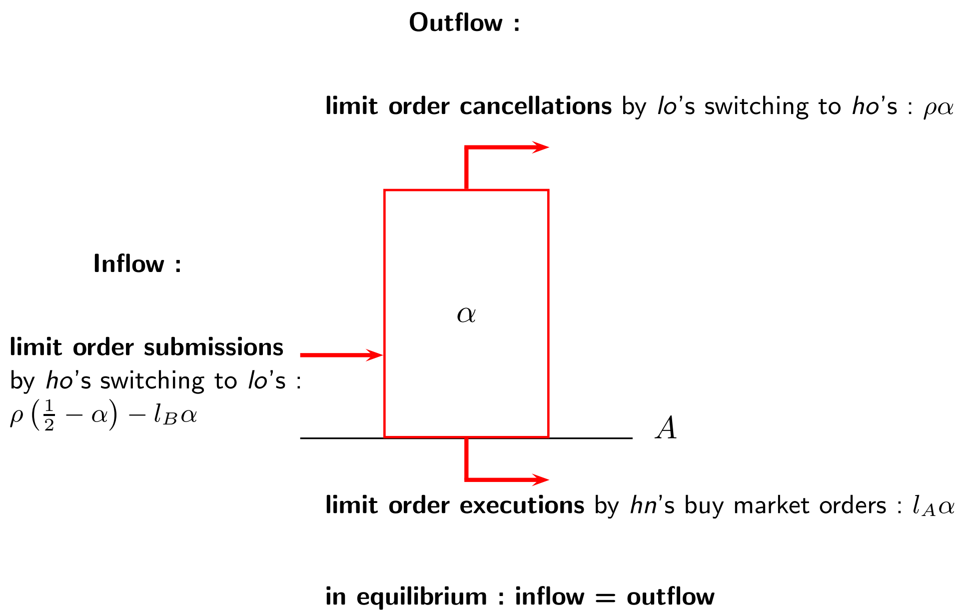

Figure 3: Steady-state dynamics of market depth.

Micro-level dynamics of the limit order book. In the steady state, inflows and outflows of limit orders must - on each side of the order book - exactly compensate each other. Figure 3 shows all order flows on the ask side in the steady-state condition. The inflow formula requires a one-step calculation. The flow of investors who become *lo* and then place sell limit orders is equal to $(1 - m_B)\rho\left(\frac{1}{2} - \alpha\right)$. It can be replaced by $\rho\left(\frac{1}{2} - \alpha\right) - l_B\alpha$ because, by equation (11), the flow of sell market orders is equal to the flow of buy limit order executions: $m_B\rho\left(\frac{1}{2} - \alpha\right) = l_B\alpha$. Hence the steady-state condition for the ask side is

$$\rho\left(\frac{1}{2} - \alpha\right) - \rho\alpha - l_A\alpha - l_B\alpha = 0. \qquad (19)$$

The same reasoning applies on the bid side and yields exactly the same steady-state condition. Therefore, market depth α depends on the limit order execution rate as follows:

$$\alpha = \frac{1}{2}\frac{\rho}{2\rho + l_A + l_B} \qquad (20)$$

Proposition 4 can now be used to deduce the equilibrium market depth and the equilibrium mixed-strategy parameters.

Proposition 5. *In the steady state without news arrival, the equilibrium market depth and the equilibrium mixed strategy parameters are, respectively,*

$$\alpha^* = \frac{1}{2}\frac{\rho\Delta}{\delta - r\Delta}, \text{ and } m_A = m_B = m^* = \frac{1}{2}\frac{\delta - (r+2\rho)\Delta}{\delta - (r+\rho)\Delta} \tag{21}$$

3.4 Converging towards the steady state without news

Here I describe the market dynamics after prices have adjusted to news. In this phase, the market converges toward the steady state without news that was described in Section 3.3.

During this last phase, the evolution of investors' populations is determined by the function, $\alpha(t)$, as follows (see Proposition 1),

$$L_{hn}(t) = L_{lo}(t) = \alpha(t), \; L_{ho}(t) = L_{ln}(t) = \frac{1}{2} - \alpha(t) \tag{22}$$

Let $\alpha_{residual}$ denote the value of the free parameter, defined in Proposition 1, at the end of the transition phase. This value is such that, at $t = \tau + T$, $\alpha(t) = \alpha_{residual}$. The parameter $\alpha_{residual}$ is endogenized in section 3.5 where the transition phase is described.

Market dynamics. Once the transition phase is over, all investors of types lo and hn can resubmit their limit orders at the new best bid and ask prices B^u and A^u (after positive news) or B^d and A^d (after negative news), if needed.

At the beginning of the last phase, after positive news, all investors of types lo have already resubmitted their order at price A^u during the transition phase, and wait for order execution. Consequently, at time $t = \tau + T$, market depth at price A^u is equal to $\alpha_{residual}$, and is equal to $\alpha(t)$ further on. In contrast, at the beginning of the last phase, all investors of types hn are investors who have not yet reacted to news, and whose order is still at the former bid price. During the last phase, these orders are canceled after investors private valuations change, at rate ρ, or resubmitted at price B^u, at rate λ. Consequently, during the last phase, the mass of investors of types hn with stale limit orders decays, and is equal to $\alpha_{residual}e^{-(\rho+\lambda)(t-\tau-T)}$. Moreover, market depth at price B^u is equal to $\alpha(t) - \alpha_{residual}e^{-(\rho+\lambda)(t-\tau-T)}$. The analysis is symmetric for negative news.

For simplicity, I denote the best bid and ask prices, B and A. Under the approximation that λ is very large, the resubmission process is extremely rapid, and the stock of stale orders is depleted almost instantaneously. So, as a first-order approximation, I can assume that market depths at prices A and B are both equal to $\alpha(t)$ during the last phase[7].

Strategy. The equilibrium strategy is similar to the one in the steady state without news. As in Proposition 3, investors of type ho or ln do not submit orders. At the same time, investors of type lo or hn are indifferent between limit and market orders; with probability $m(t)$ they use a market order. Because there is no news arrival, bid and ask prices do not change. As in Section 3.3, the investor's trade-off is still between a limit order at A (or B) and a market order at B (or A). Furthermore, this trade-off is not affected by changes in the market depth. Hence the limit order execution rate must equal that in the steady state, l^*, as defined in Proposition 4. The value functions are also unchanged, and the relation between $m(t)$ and the equilibrium execution rate l^* is the same as in equation (10):

$$l^* = \frac{m(t)\rho\left(\frac{1}{2} - \alpha(t)\right)}{\alpha(t)}. \tag{23}$$

The flows of orders going in and out of the order book are the same as in Figure 3 - except that the net inflow is no longer zero. More specifically, the net inflow corresponds to the instantaneous change in the market depth, which is denoted $d\alpha$ and defined as

$$\frac{d\alpha}{dt} = \rho\left(\frac{1}{2} - \alpha(t)\right) - \rho\alpha(t) - 2l^*\alpha(t) \tag{24}$$

Equation (24) shows that the dynamics of $\alpha(t)$ follows an ordinary differential equation of degree 1. The solution to this equation is[8]

$$\alpha(t) = \alpha^*\left(1 - e^{-2(\rho+l^*)(t-\tau-T)}\right) + \alpha_{residual}e^{-2(\rho+l^*)(t-\tau-T)}. \tag{25}$$

Thus, market depth $\alpha(t)$ converges towards its steady state without news, α^*.

[7]In Section C of the Online Appendix, I provide a comprehensive analysis of the last phase. Section 3.4 focuses on the limit case where $\lambda \to \infty$.

[8]Accounting for orders that initially are not at the best bid and ask prices, and that are resubmitted, would add a third term to equation (25) that decays at rate λ; see online appendix C.2.

3.5 The Transition phase

Upon news arrival at $t = \tau$, the asset value changes and the transition phase begins. At the start of this phase, the bid and ask prices are B and A while the market depth, α on both sides, is inherited from the initial steady state (α is endogenized in Section 3.6). The endogenous duration of the transition phase is T. At the end of this phase, trading prices adjust either to A^u and B^u or to A^d and B^d.

Transition phase strategy. Upon news arrival, if the new asset value is $v + \omega$ then investors of types hn and ln would like to pick off any stale sell limit order, at price A, left by type-lo investors. Conversely, investors of type lo do *not* want their order to be picked off by the hn and ln types. According to Assumption 1, investors do not react to news at the same instant. Those who contact the market before the end of the transition phase can either exploit stale limit orders or cancel those orders to avoid being picked off. The next proposition describes investor strategies during the transition phase.

Proposition 6. *Following news arrival, investors who contact the market during the transition phase play one of the following strategies:*

- *If the new asset value is $v + \omega$: types lo cancel their limit orders (if any) at price A and submit a new limit order at price A^u; types ho cancel any sell limit order and stay out of the market; types ln cancel any buy limit order, send a buy market order, and immediately follow the type-lo strategy; types hn cancel any buy limit order, send a buy market order, and immediately follow the type-ho strategy.*
- *If the new asset value is $v - \omega$: types hn cancel their limit orders (if any) at price B and submit a new limit order at price B^d; types ln cancel any buy limit order and stay out of the market; types ho cancel any sell limit order, send a sell market order, and immediately follow the type-hn strategy; types lo cancel any sell limit order, send a sell market order, and immediately follow the type-ln strategy.*

Order book dynamics in the transition phase. Proposition 6 states that, during the transition phase, trading occurs only on one side of the order book - namely, the side featuring profit opportunities. If the new asset value is $v + \omega$, then trading occurs only at price A; if that new value is $v - \omega$, then trading occurs only at price B. The transition phase ends when market depth D_A or D_B is equal to zero.

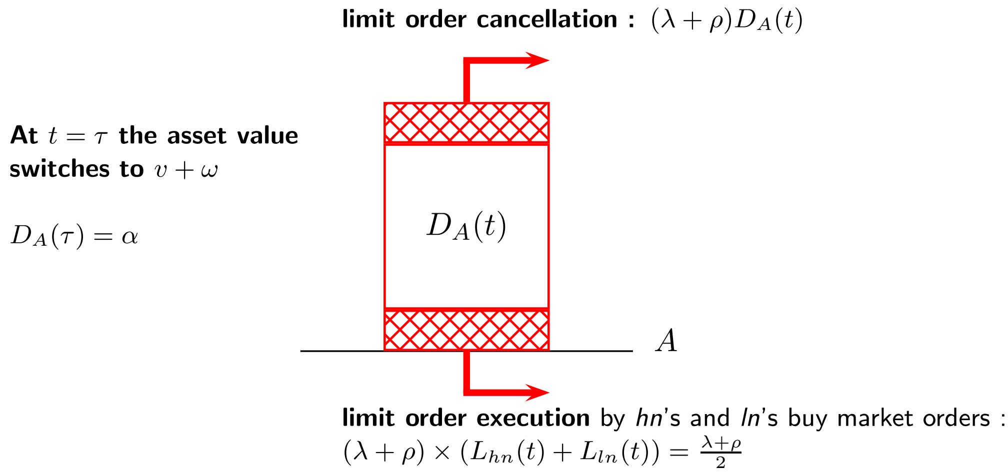

Figure 4: Dynamics of market depth: At the ask price following arrival of positive news.

As illustrated in Figure 4, if the new asset value is $v+\omega$ then market depth D_A decreases, which reflects two underlying processes. At each instant t, the flow of type-lo investors - with a limit order at price A and who contact the market and cancel their order - is equal to $(\lambda + \rho)D_A(t)$. At the same time, the flow of non-asset-owning investors, hn and ln, who contact the market and send buy market orders that execute at price A is equal to $(\lambda + \rho)(L_{hn}(t) + L_{ln}(t)) = (\lambda + \rho)/2$. As a consequence, the dynamics of D_A is ruled by the following first-order differential equation:

$$\frac{dD_A}{dt} = -(\lambda + \rho)D_A(t) - \frac{\lambda + \rho}{2} \tag{26}$$

If the new asset value is instead equal to $v - \omega$, then the dynamics of market depth D_B is ruled by the same differential equation. The following proposition gives the solution to that equation.

Proposition 7. *During the transition phase, if the new asset value is* $v+\omega$*, then the market depth dynamics at price* A *is*

$$D_A(t) = D(t) = -\frac{1}{2} + \left(\alpha + \frac{1}{2}\right) e^{-(\rho+\lambda)(t-\tau)}. \tag{27}$$

If the new asset value is $v - \omega$*, then the market depth dynamics at price* B *is likewise* $D_B(t) = D(t)$*. The duration* T *of the transition phase in both cases is such that* $D(\tau+T) = 0$*, and it can be written as*

$$T = \frac{\ln(1+2\alpha)}{\rho+\lambda}. \tag{28}$$

The transition phase dynamics determines the populations' sizes of investors with types lo and hn at the beginning of the third phase, which are denoted $\alpha_{residual}$ in Section 3.4. Consider the population's dynamics of type-hn investors when the new asset value is $v+\omega$. During the transition phase, no new investor becomes type hn, and all investors who are of type hn and also contact the market end up buying the asset. In particular, a type-ln investor who becomes hn also contacts the market at the same time and then immediately becomes an ho type after sending a buy market order. Moreover, types ho stay out of the market; types lo who contacted the market before being picked off have a limit order at price A^u and do not trade before the end of the transition phase; and type-lo investors whose orders were executed become type ln. The market contact rate of type-hn investors during the transition phase is $\rho+\lambda$. So on the time interval $[\tau, \tau+T]$, the mass of hn types declines in the following way:

$$L_{hn}(t) = \alpha e^{-(\rho+\lambda)(t-\tau)} \tag{29}$$

Hence it follows from Proposition 1 that $\alpha_{residual}$ can be calculated as $L_{hn}(\tau+T)$. Therefore,

$$\alpha_{residual} = \frac{\alpha}{1+2\alpha}. \tag{30}$$

Value functions. During the transition phase, value functions become time-dependent. When giving the Bellman equations that define value functions, one is actually writing a system of linear differential equations.

Consider the case where the new asset value is $v+\omega$. A type-lo investor with a limit order at price A is denoted $lo-A$. As soon as he contacts the market, a $lo-A$ investor cancels his limit order and then submits a new limit order at price A^u; in this way he becomes a $lo - A^u$ investor (or becomes an ho type if he contacts the market following a change in private valuation). However, his order may have been executed in the meantime - in which case he would have become a type ln and would have received the payment A.

The limit order execution rate at price A is equal to the buy market orders flow, which in turn is equal to $(\lambda + \rho)/2$ divided by the depth $D(t)$. This execution rate is denoted $k(t)$; formally, we have

$$k(t) = \frac{\lambda + \rho}{2D(t)}, \tag{31}$$

The value function of a $lo - A$ investor can then be derived as follows:

$$(r+\rho+\lambda+k(t))V^u_{lo-A}(t) = r(v+\omega)-\delta+\frac{\partial V^u_{lo-A}}{\partial t}+\rho V^u_{ho}(t)+\lambda V^u_{lo-A^u}(t)+k(t)(V^u_{ln}(t)+A) \tag{32}$$

In order to solve for the differential equations system, I need to determine boundary conditions. These are obtained using the continuation values prevailing when the transition phase ends. At time $\tau + T$, if the investor still has a limit order at price A then her order is executed with certainty. She then receives the payment A and the continuation value of a type-ln investor at the beginning of the last phase - that is, the value function in the steady state without news when the asset value is $v + \omega$. This function is denoted $\bar{V}^u_{ln}$. Hence the boundary condition can be written as $V^u_{lo-A}(\tau + T) = \bar{V}^u_{ln} + A$. Bellman equations for types ho, ln, hn, and $lo - A^u$ are derived in the proof of Proposition 6.

3.6 The Initial steady state

In this section I describe the initial steady state - that is, before news arrives. This state is similar to the one described in Section 3.3; the only difference is that investors anticipate news arrival at horizon τ.

Steady state strategy. As in Proposition 3, types ho and ln stay out of the market while types lo and hn play a mixed strategy m when choosing between limit and market orders. I define l, the limit order execution rate, as

$$l = \frac{m\rho\left(\frac{1}{2} - \alpha\right)}{\alpha}. \tag{33}$$

Unlike the steady state without news, the initial phase may stop before game ends, and prices may move before the asset pays-off. Yet, investors don't place order deep in the book. Investors don't place sell orders at prices strictly in between A and A^u (resp. buy orders strictly in between B^d and B) because during the initial steady state these orders have no chance to be executed. They would be executed only following positive news (resp. negative) but this would be at a lower (resp. larger) than the new equilibrium price A^u (resp. B^d).

Neither do investors place orders at future equilibrium prices A^u or B^d. For instance, an investor of type ho does not find it optimal to place a sell order at price A^u because he knows that he may trade at this price only during the last phase of the game and only after positive news. During that phase, however, it is optimal only for type-lo investors to have a sell order in the book. An investor ln likewise does not find it optimal to place a sell order at price B^d. Types lo and hn do neither find it optimal to submit orders at A^u or B^d. In principle, they may. This scenario could be obtained if limit orders at best prices were waiting long enough before execution. Yet, I consider a situation where ω is large enough ($\omega > \widehat{\omega}(\delta, \Delta)$, see Assumption 5) so that the limit order execution rate is large, and this equilibrium does not arise.

Value functions. As compared with the value functions derived in Section 3.3, value functions in the initial steady state incorporate a new term: the average continuation value at the beginning of the transition phase as weighted by μ, the news arrival intensity. For instance, an investor of type ho has a new value function $V_{ho}^u(\tau)$ at the beginning of the transition phase if the new asset value is $v+\omega$. I relabel this continuation value $\widehat{V}_{ho}^u$. Similarly, her new value function is $\widehat{V}_{ho}^d$ if the new asset value is $v-\omega$. Following the arrival news, the average continuation value of a type-ho investor is $\left(\widehat{V}_{ho}^u + \widehat{V}_{ho}^d\right)/2$. So in comparison with her value function without news (as defined by equation (12)), an ho type's value function has an additional term:

$$(r + \rho + \mu)V_{ho} = rv + \rho V_{lo} + \mu \frac{\widehat{V_{ho}^u} + \widehat{V_{ho}^d}}{2} \tag{34}$$

One can similarly rewrite the type-hn investor's value function as

$$(r + \rho + l + \mu)V_{hn} = \rho V_{ln} + l(V_{ho} - B) + \mu \frac{\widehat{V_{hn}^u} + \widehat{V_{hn}^d}}{2}. \tag{35}$$

As explained in Section 3.3, a type-hn investor uses equation (14) to compare the difference in continuation values (conditional on any realization of the next event) between using a limit order and a market order net of the payment A - and then takes the expected difference into account. Unlike the case without news, the next event can also be a news arrival. Conditional on good news and bad news, the difference in continuation values between placing a market order (net of the payment A) and placing a limit order are, respectively, $\widehat{V_{ho}^u} - A - \widehat{V_{hn}^u}$ and $\widehat{V_{ho}^d} - A - \widehat{V_{hn}^d}$. Hence the unconditional difference in continuation value, $V_{ho} - A - V_{hn}$, is as follows:

$$(r+\rho+l+\mu)(V_{ho}-A-V_{hn}) = r(v-A)+\rho(V_{lo}-A-V_{ln})+l\Delta+\mu\frac{\widehat{V_{ho}^u} - A - \widehat{V_{hn}^u} + \widehat{V_{ho}^d} - A - \widehat{V_{hn}^d}}{2}. \tag{36}$$

Just as before, the indifference conditions for investors of types hn and lo are (respectively) $V_{hn} = V_{ho} - A$ and $V_{lo} = V_{ln} + B$. Incorporating these equilibrium conditions into equation (36) allows one to compute the limit order execution rate l, which is a function of market depth α. Formally,

$$l(\alpha) = \frac{1}{\Delta}\left[\frac{\delta - (r+2\rho)\Delta}{2} + \mu\frac{\widehat{V_{ho}^u} - A - \widehat{V_{hn}^u} + \widehat{V_{ho}^d} - A - \widehat{V_{hn}^d}}{2}\right] \tag{37}$$

Market depth, α, enters this expression through the last term, which is the average difference in continuation values between placing a market order (net of the payment A) and placing a limit order due to news arrival. As a consistency check, one can see that $l = l^*$ for $\mu = 0$.

To solve for the equilibrium values of l and α, I use the steady-state condition given by equation (20):

$$\alpha = \frac{\rho}{4(\rho + l(\alpha))}. \tag{38}$$

In equilibrium, the limit order execution rate is positive; hence equation (38) implies that α must be less than 1/4. Two other conditions must be verified. Namely, types lo and hn must prefer submitting orders at prices A and B (respectively) to submitting them at prices A^u and B^d. This equilibrium condition is derived in the proof of Proposition 8.

Proposition 8. *The equilibrium market depth is the unique solution of equation (38) on the interval* $[0, 1/4]$. *The equilibrium limit order execution rate is* $l_{eq} = l(\alpha_{eq})$.

In the rest of this section I investigate the case of a negligible reaction delay, as when $\lambda \to \infty$. This special case allows me to obtain closed-form solutions for α_{eq} and l_{eq}. At the limit, the transition phase becomes practically instantaneous. As λ approaches infinity, the investor's ability to react quickly to news increases but, simultaneously, competition from other investors becomes more intense. Thus limit orders are not only cancelled at a faster rate but also picked off at a faster rate. For any λ, one can compute the probability $p(\alpha)$ that, following news arrival, a given investor contacts the market before the end of the transition phase:

$$p(\alpha) = 1 - e^{-(\rho+\lambda)T} = 1 - \frac{1}{1+2\alpha} \tag{39}$$

The probability of reacting "fast", and thus of being able to exploit new information, does not depend on λ. Equation (39) quantifies the intuition that whether investors are all fast or all slow makes no difference in a race triggered by news arrival.

The equilibrium can be solved in closed form under the approximation $\lambda \to \infty$, as the next proposition shows.

Proposition 9. *When $\lambda = \infty$ and for a given α, the differences in continuation values between placing a market order (net of payment A) and placing a limit order, conditional on news arrival, are as follows:*

$$\widehat{V^u_{ho}} - A - \widehat{V^u_{hn}} = \frac{1}{1+2\alpha}\omega, \tag{40}$$

$$\widehat{V^d_{ho}} - A - \widehat{V^d_{hn}} = -\Delta. \tag{41}$$

The equilibrium market depth is the unique solution, on the interval $[0, 1/4]$, to the equation

$$2\alpha \left[\delta - r\Delta + \mu\left(\frac{\omega}{1+2\alpha} - \Delta\right)\right] = \rho\Delta. \tag{42}$$

Market depth, α_{eq}, has a closed-form formula that is left in the proof of the proposition (see equation (73)). When μ or ω goes towards $+\infty$, α_{eq} goes towards 0.

Equation (40) can be understood by comparing the situations of (a) an investor who pays A before news arrival and thus becomes an *ho* type and (b) an *hn* type who has submitted a buy limit order at price B that has not been executed prior to news arrival. When the new asset value is $v + \omega$, a type-*hn* order at price B has no chance of being executed. This investor would like to buy the asset at price A before the transition phase ends; however, he fails to do so with probability $1 - p(\alpha)$. The investor who paid A to become ho is already in the optimal situation sought by the investor of type hn. It follows that submitting a limit order generates a cost of news related non-execution risk, $\widehat{V^u_{ho}} - A - \widehat{V^u_{hn}}$. This cost is equal to the difference between the new and the former asset value, ω, multiplied by $1 - p(\alpha)$, the probability of *not* contacting the market before the transition phase ends.

To explain equation (41), I compare the same two investors when the new asset value is instead $v - \omega$. The type-hn investor would like to cancel his limit order at price B and then resubmit it at price B^d, thereby becoming a $hn - B^d$ investor; he succeeds in this endeavor with probability $p(\alpha)$. Otherwise his order is picked off, upon which he is paid B and becomes an ho type. The type-ho investor who has already paid A would like to sell the asset at price B and hence become a $hn - B^d$ investor; he either succeeds (with probability $p(\alpha)$) or remains an investor of type ho. In sum, with probability $p(\alpha)$ both investors become $hn - B^d$ and with probability $1 - p(\alpha)$ both investors become ho. The only difference between these two outcomes is in the overall net payments. The investor who is initially of hn type pays $(1 - p(\alpha))B$ on average; whereas the investor who initially paid A to become ho pays, on average, $A - p(\alpha)B$. Hence $\widehat{V^d_{ho}} - A - \widehat{V^d_{hn}} = B - A = -\Delta$, which implies that- conditional on the new asset value being $v - \omega$ - the investor is better-off placing a buy limit order, despite the associated picking-off risk, than placing a market order.

Under Assumptions 4 and 5, the difference in continuation values between a buy (resp. sell) market order and a buy (resp. sell) limit order, conditional on negative (resp. positive) news, is small compared to the one conditional on positive (resp. negative) news. As a consequence, the difference in continuation values between a limit and a market order is a relative adverse selection cost that is mainly due to non-execution risk following news arrival.

4 Empirical implications

This section develops the model's empirical implications. After detailing my approach in Section 4.1, in Sections 4.2 - 4.4 and Section 4.6 I derive cross-sectional predictions - plotted in the five graphs of Figure 5 - that are based on comparative statics with respect to the news arrival frequency μ. In Section 4.5 I derive time-series predictions, plotted in the six graphs of Figure 6, that focus on the joint dynamics of order submissions and prices.

4.1 Methodology

The aim of this section is to explain the assumptions under which I work when deriving empirical predictions. I also explain which type of explanatory and control variables the econometrician should use to test such empirical predictions.

Throughout Section 4, I assume that λ is much larger than either the news arrival intensity, μ, or the idiosyncratic shock intensity, ρ. Therefore, as a first-order approximation I assume that α_{eq} and l_{eq} are equal to their respective values given in Proposition 9. I also assume implicitly that order book trading and quoting activity can be precisely observed during even an extremely short transition phase.

Controlling for unconditional price variability. One may test for the effect of the news arrival frequency by using a news database to count the number of days (or hours) with and without news, thereby constructing the explanatory variable. In such testing it would be crucial to control for other model parameters, especially the jump size, ω. The jump size could be measured empirically as the average absolute price jump around news events. Since the parameter ω is likely difficult to estimate, econometricians might prefer to control for volatility, generally speaking, and specifically here for price variability - that is, the variance of price changes (as in Tauchen and Pitts 1983).

In my model, the unconditional price variability is the product of the variance of the change in price conditional on news, ω^2, and the probability of news arrival during the game. Hence I define the unconditional price variability, σ^2, as follows:

$$\sigma^2 = \frac{\mu}{r + \mu} \times \omega^2. \tag{43}$$

In the rest of Section 4 I make predictions of the following form. I compare a given outcome (depth, price adjustment delay,...) between two stocks with the same unconditional price variability but where one stock experiences a higher news frequency. Therefore, predictions concerning the jump size are unnecessary. Indeed, equation (43) shows that if σ is fixed then μ and ω are inversely related. So when a model outcome increases with the news frequency, it necessarily decreases with the jump size.

The price variability in my model is generated by the unscheduled news process. In reality, it could be generated also by such sources as privately informed traders, inventory management by market makers, and other types of price pressures. It would be ideal to control for these factors so that the model's predictions can be properly tested.

Controlling for a size effect. The model's current design does not accommodate a size effect, as follows from my assumption of a unit mass of investors. In the general case, the mass of investors is equal to some M. More importantly, all extensive outcomes (e.g., market depth, trading volume, flow of order submissions) should be multiplied by M. An obvious approach to addressing this issue empirically would be to identify a proxy for the investors' base size and then to control for that variable in empirical tests. However, I approach the problem by deriving - in addition to the first prediction - another prediction generated by the same mechanism but using either an intensive outcome or a ratio of two extensive outcomes.

4.2 Market depth and limit order execution rate

Other things equal, if the news frequency, μ, or the jump size, ω, increases then investors are less willing to place limit orders. Indeed, one can use equations (37), (40), and (41) to show that the equilibrium execution rate (l_{eq}) increases so as to keep investors indifferent between limit and market orders. As a result, the market depth, α_{eq}, must also decrease (see equation (38)).

This result is not surprising. It is well known that volatility has a negative effect on the provision of liquidity. In order to develop novel predictions regarding the effect of unscheduled news on liquidity, I employ the methodology described in Section 4.1. The following proposition relates the equilibrium market depth, α_{eq}, to the news frequency, μ, while holding constant the unconditional price variability, σ^2.

Proposition 10. *Keeping unconditional variability, σ^2, constant, market depth decreases with news arrival frequency.*

This proposition shows that unconditional price variability is not the sole determinant of the trade-off between limit and market orders. For a given level of unconditional price variability, an asset value that changes more frequently - but by a smaller amount each time - will have less market depth than the opposite case; see panel (a) of Figure 5. The intuition is that, if the unconditional price variability is held constant, then the expected absolute change in price, or $\mu\omega/(r+\mu)$, increases with the news arrival frequency. Hence the adverse selection cost of a limit order increases.

Prediction 1. *Compare two stocks with same unconditional price variability, then the stock with the higher news arrival frequency has the lower market depth.*

As mentioned in Section 4.1, if investors' base were not normalized then market depth would depend on the number of investors. Alternatively, I consider the limit order execution rate, l_{eq}. This rate is an intensive outcome (a "per order" quantity) and so does not depend on the investors' base. Panel (b) of Figure 5 shows that limit order execution rate depends positively on news frequency. The limit order execution rate can be estimated empirically as the inverse of the average time to execution of limit orders at the best bid and ask prices.

Prediction 2. *Compare two stocks with same unconditional price variability, then the stock with the higher news arrival frequency has the higher limit order execution rate.*

4.3 Duration between news arrival and price changes

In equilibrium, after news arrives the adjustment of prices occurs once all stale limit orders that offered a profit opportunity have disappeared through either execution or cancellation. The duration between news arrival and the adjustment of prices is the duration of the transition phase, denoted T. This variable is a measure of price discovery and can be written as

$$T = \frac{\ln(1+2\alpha_{eq})}{\rho+\lambda} \tag{44}$$

Equation (44) shows clearly that, when the market is not deep, duration is short. Hence the higher the news arrival frequency, the faster the price adjustment to news. Since the news arrival frequency, μ, has a negative effect on the initial market depth, α_{eq}, it follows that the mass of stale orders executed or canceled during the transition phase is lower and hence that the transition phase is shorter. The next prediction, which summarizes this finding, is illustrated in panel (c) of Figure 5.

Prediction 3. *Compare two stocks with same unconditional price variability, then the stock with the higher news arrival frequency has the shorter transition phase duration.*

A proper test of Prediction 3 requires that one controls for λ. If λ is interpreted as the trading system's latency, then it could vary across assets for several reasons. For example, there may be differences in electronic trading platform infrastructures or in message traffic volume relative to available bandwidth.

4.4 Trades and cancellations

In this section I derive implications concerning the dynamics of trades and cancellations. In particular, I analyze the relative importance of trades and cancellations in explaining the flow of limit orders that leave the order book. Excessive order cancellations are scrutinized by regulators, who view such excess as a possible indicator of manipulative quoting activity by high-frequency traders. Nonetheless, quoting dynamics can also reflect fundamental factors. This paper argues that the unscheduled news process is such a factor.

I focus first on the effect of news. Trades due to news correspond to the execution of stale limit orders during the transition phase. Limit order cancellations due to news correspond to (i) limit orders that avoid being picked off during the transition phase and (ii) limit order cancellations on the other side of book. The following proposition gives the number of trades during the transition phase.

Proposition 11. *During the transition phase, the numbers of trades is equal to*

$$tr(\alpha_{eq}) = \frac{\ln(1+2\alpha_{eq})}{2} \tag{45}$$

Equation (45) can be used to deduce that the amount of limit orders canceled to avoid being picked off is equal to $\alpha_{eq} - tr(\alpha_{eq})$, simply because execution and cancellations add up to market depth. On the other side of the book, all limit orders end up being canceled; then the corresponding amount of cancellations is α_{eq}. Overall, the outflow of limit orders due to news come to $2\alpha_{eq}$. Hence I can define the trades' share in the orders' outflow due to news, TS_{news}, as the ratio of trades to the sum of trades and cancellations in the transition phase:

$$TS_{news} = \frac{\ln(1+2\alpha_{eq})}{4\alpha_{eq}}. \tag{46}$$

The following corollary analyzes how the share TS_{news} depends on the frequency with which news arrives.

Corollary 1. *The share, TS_{news}, is an increasing function of news arrival frequency, μ.*

The intuition behind this result is as follows. The population of investors who can take advantage of stale orders has a fixed size: it corresponds to the group of non-owners when the asset value goes up (since they can buy the asset) and to the group of owners when the asset value goes down (since they can sell the asset). Each group is of the same size, 1/2. In the transition phase, the market contact rate of an investor is equal to $\lambda + \rho$. Therefore, the flow of market orders that execute against stale limit orders is equal to $(\lambda + \rho)/2$ and does not depend on either μ or ω. The contact rate of investors who want to cancel their order is $\lambda + \rho$. At the beginning of the transition phase, the mass of investors who want to cancel is α_{eq} on both side of the market. Hence the flow of limit order cancellations is a positive function of α_{eq}. So if news is more frequent then the market depth α_{eq} is less; in this case, the flow of limit order cancellations is also lower but the flow of market orders is the same. Thus the trades' share in the orders' outflow is larger, as formally predicted next and as illustrated in panel (d) of Figure 5.

Prediction 4. *Compare two stocks with same unconditional price variability, then the stock with the higher news arrival frequency has the higher TS_{news} share.*

In order to study the dynamics of trades' share in the orders' outflow, I study what happens in the initial steady state. In that steady-state phase, a limit order either is executed at Poisson intensity l_{eq} or is canceled, when the agent's private valuation changes, at Poisson intensity ρ. Therefore, at each instant, the amount of trades and cancellations are (respectively) $2l_{eq}\alpha_{eq}$ and $2\rho\alpha_{eq}$. And then, I can use equation (38) to derive the trades' share in the orders' outflow in the initial steady state, TS_{steady}, as

$$TS_{steady} = \frac{l_{eq}}{\rho + l_{eq}} = 1 - 4\alpha_{eq}. \tag{47}$$

The following proposition compares the trades' share in the orders' outflow due to news with the trades' share in the orders' outflow in the steady state by analyzing their ratio: $RTS = TS_{news}/TS_{steady}$.

Proposition 12. *The ratio of the trades' shares, RTS is equal to*

$$RTS = \frac{\ln(1 + 2\alpha_{eq})}{4\alpha_{eq}(1 - 4\alpha_{eq})}. \tag{48}$$

The RTS ratio decreases with news arrival frequency, μ.

When the frequency of news arrival increases, the limit order equilibrium execution rate also increases. Hence limit orders are less likely to be canceled following a changed private valuation, which means that TS_{steady} then decreases. At the same time, the market depth is lower and so TS_{news} is lower as well. The overall effect is not clear-cut. When μ is large, α_{eq} approaches 0 and then RTS converges to 1/2; it follows that TS_{steady} is greater than TS_{news}. When μ is low, however, RTS can be either greater or less than 1 depending on the values of other parameters.

More interesting is Proposition 12's implication that the RTS should vary across stocks. This claim is reiterated by the following prediction and is portrayed in panel (e) of Figure 5.

Prediction 5. *Compare two stocks with same unconditional price variability, then the stock with the higher news arrival frequency has the lower RTS ratio.*

The theoretical literature on quoting activity is sparse. However, Yueshen (2014) shows that if market makers are uncertain of their position in the queue for liquidity provision, then in equilibrium they cancel and resubmit their orders several times before execution. Thus there is a sense in which quoting activity is excessive under that setup. In contrast, my model shows that unscheduled news is a source of stochastic variation for quoting activity as measured by the trades' share in the orders' outflow.

4.5 Order book imbalances

Prior to news arrival, trading occurs because of differences in private valuations - though at prices generally in line with the asset value. Yet when news arrives, trading prices no longer accord with the new asset value. This mismatch generates imbalances, in both the order book and the order flows, that disappear once prices have adjusted. Consider the difference between the market depth at the best ask and at the best bid; I refer to that difference as the *depth imbalance* and denote it $DEimb_{A/B}$. There is no such imbalance prior to news arrival. Upon news arrival and during the transition phase, if the new asset value is $v + \omega$ then the depth at price A is equal to $D(t)$ and the depth at price B is equal to $\alpha_{eq} e^{-(\rho+\lambda)(t-\tau)}$, as shown by equation (29). Hence the depth imbalance between the ask side and the bid side can be written as

$$DEimb_{A/B}(up) = D(t) - \alpha_{eq} e^{-(\rho+\lambda)(t-\tau)} = -\frac{1}{2}\left(1 - e^{-(\rho+\lambda)(t-\tau)}\right). \qquad (49)$$

However, if the new asset value is $v - \omega$ then the depth imbalance has the opposite value: $DEimb_{A/B}(down) = -DEimb_{A/B}(up)$. The intuition underlying this result is that stale limit orders that offer a profit opportunity disappear more rapidly because on this side they are both canceled and executed whereas, on the other side, they are only canceled. After the transition phase, the order book imbalance is nil once again.

One can similarly define the *market order imbalance*, $MOimb_{A/B}$. This term represents the difference between the flow of buy market orders and sell market orders (at each instant) and the *cancellation imbalance*, $CAimb_{A/B}$, which is the difference (at each instant) between the flow of cancellations at the best ask and the best bid. These imbalances are both nil outside the transition phase. During that phase, however, they have the following expressions:

$$MOimb_{A/B}(up) = -MOimb_{A/B}(down) = \frac{\rho + \lambda}{2}, \tag{50}$$

$$CAimb_{A/B}(up) = -CAimb_{A/B}(down) = (\rho + \lambda)DEimb_{A/B}(up). \tag{51}$$

These imbalances anticipate a future price change because they are indicative of a transition phase, which in turn is associated with a change in price level at play. The next prediction then follows, as shown in panels (c), (d), and (e) of Figure 6.

Prediction 6. *Positive (resp. negative) market order imbalance, and negative (resp. positive) depth and cancellation imbalances predict a positive (resp. a negative) change in price.*

Cont, Kukanov, and Stoikov (2014) analyze the price impact of order flow imbalances. They define the order flow imbalance (OFI) as the difference between "demand" and "supply" orders. In my framework, their OFI would be defined as $OFI = MOimb_{A/B} - CAimb_{A/B}$; thus $OFI(up) = e^{-(\rho+\lambda)(t-\tau)}/2$. My model predicts that a positive OFI precedes a positive change in price. Cont and colleagues use order book data (at 10-second intervals) to test for a linear dependence of the change in price on the OFI, and they find a positive and significant relation. This result is consistent with Prediction 6.

Since imbalances anticipate a change in price, it follows that they could be exploited - especially by algorithmic trading strategies. Cartea, Donnelly, and Jaimungal (2015) document the predictive power of order book imbalances for future price movements on the Nasdaq exchange. Kwan and Philip (2015) show that high-frequency traders (HFTs) on the Australian Securities Exchange do, in fact, take advantage of this predictability. These authors report that HFTs are more successful than non-HFTs at trading ahead of large favorable depth imbalances and at canceling their orders in the face of large unfavorable imbalances.

4.6 Trading volume and price variability

In this section, I study the relationship - between trading volume and price variability - that is implied by the model.

High frequency relationship. The trading volume-price variability relationship is most evident at high trading frequencies. Before news arrival, prices do not change and the trading intensity (i.e., trading volume per infinitesimal time interval dt) is low: only $2l_{eq}\alpha_{eq}$. Following news arrival and during the transition phase, there is a sharp increase in trading intensity - that is, in the arrival rate $(\rho + \lambda)/2$ of market orders (see Section 4.4). When the transition phase ends, the price level jumps; at this instant, the price variability is ω^2. The market then converges to a new steady state, in which prices do not change and the trading intensity is low (it now equals $2l^*\alpha(t)$). This time series reveals that an increase in trading volume predicts a future change in price; see panel (b) of Figure 6.

Low frequency relationship. The relation between trading volume and price variability can also be seen at low frequencies (i.e., across trading games). Indeed, with probability $r/(r + \mu)$ the trading game ends before news arrival. This is what happens on a no-news day (or hour) during which there is no price change. With probability $\mu/(r + \mu)$, the game ends after news arrival; this outcome occurs on a no-news day (or hour) during which the price variability is ω^2. As compared with a no-news period, news generates additional trading volume during the transition phase - what I call the "extra trading volume due to news". This extra volume is equal to $\ln(1 + 2\alpha_{eq})/2$ (see Proposition 11). So across trading games, the price variability and the extra trading volume due to news are two binomial random variables that are perfectly correlated. The covariance of these two variables is positive[9].According to equation (43), the covariance can be written in two forms according as whether the jump size ω or the unconditional price variability σ^2 is held constant:

[9]Let us consider two binomial random variables, a and b such that with proba. $1 - p$, $a = a_0$, $b = b_0$, and with proba. p, $a = a_0 + \Delta a$, $b = b_0 + \Delta b$, then $var(a) = p(1 - p)\Delta a^2$, $cov(a, b) = p(1 - p)\Delta a\Delta b$, and $corr(a, b) = sign[\Delta a\Delta b] = \pm 1$.

$$Cov_{P,V} = \frac{r\mu\omega^2}{(r+\mu)^2}\frac{\ln(1+2\alpha_{eq})}{2} = \frac{r\sigma^2}{r+\mu}\frac{\ln(1+2\alpha_{eq})}{2}. \tag{52}$$

My model is consistent with numerous empirical studies that document the positive relation between price variability and trading volume (see Karpoff 1987; Gallant, Rossi, and Tauchen 1992). The model allows me to identify some possible primitives driving this phenomenon. In particular, an asset's news arrival frequency determines the variability in both the realized price and the trading volume. From equation (52) I deduce that, when the unconditional price variability is held constant, a higher frequency of news arrival reduces the covariance between trading volume and price variability. The mechanism underlying this effect is that the news arrival frequency has an unambiguously negative effect on the extra trading volume due to news because it also reduces market depth.[10] One can thus derive the following prediction, which is illustrated in panel (f) of Figure 5.

Prediction 7. *Compare two stocks with same unconditional price variability, then the stock with the higher news arrival frequency has the lower covariance between trading volume and price variability.*

As mentioned in Section 4.1, trading volume would depend on the market's size if it were not a unit mass. Using correlation instead of covariance would not help because, in my model, the former is always equal to 1. Hence that approach would fail to generate any testable predictions.

I therefore propose an alternative scale-free measure: the covariance between price variability and the trading volume as normalized by its mean. Thus I compute the mean trading volume as the sum of (a) the trading volume in the steady-state before news[11] and (b) the average trading volume associated with the news process. Trading volume in the steady state is equal to the trading intensity, $2l_{eq}\alpha_{eq}$, multiplied by the average duration of this phase, $1/(r+\mu)$. One can now use equation (38) to deduce that $4l_{eq}\alpha_{eq} = \rho(1-4\alpha_{eq})$. This equality implies that, if one controls for σ, then the steady-state trading intensity increases with the news arrival frequency. The average trading volume associated with the news process is equal

[10] In the online appendix, section G, I compute the covariance between the trading volume and the price variance by accounting for the difference in trading volume following the transition phase, in addition to the extra trading volume due to news. I find that the properties of the covariance remain unchanged.

[11] This approach makes no use of trading volume following the transition phase. In the Online Appendix (Section G, Corollary G.2) I show that accounting for this trading volume has no effect on the result.

to the probability of news arrival multiplied by the trading volume due to news, $\frac{\mu}{\mu+r}\frac{\ln(1+2\alpha_{eq})}{2}$. The volume-scaled covariance can now be written as

$$\frac{\frac{r}{\mu+r}\sigma^2\ln(1+2\alpha_{eq})}{\frac{\rho}{r+\mu}(1-4\alpha_{eq})+\frac{\mu}{\mu+r}\ln(1+2\alpha_{eq})} \tag{53}$$

One can see that, when controlling for σ^2 (or, equivalently, scaling by the average variance σ^2), the scaled covariance continues to decrease with higher μ. The intuition here is that, as the frequency of news arrival increases, investors trade more intensely before news arrival; this implication reinforces the former result.

5 Extensions

Although the model is mathematically nontrivial, it is extremely stylized and so has many limitations. In particular, there is no depth deep in the book, the bid-ask spread is always binding, there are no privately informed traders, there is no time priority, and so forth. Some of these limitations could be overcome by appropriately extending the current model, whereas successfully addressing others might well require a different type of model altogether. In this section I discuss several extensions that it seems could be implemented within the same framework.

Unscheduled vs scheduled news. The model focuses on unscheduled news. If instead the arrival of news were scheduled, so that τ is deterministic, then two consequences would follow. First, it would then be optimal for investors to cancel their limit order at the instant τ^- - that is, just before τ. Since the announcement date is known in advance, it is reasonable to assume that investors can contact the market at τ^-. In this event, the order book would become empty at that instant. Hence there would be no transition phase and, at the next instant (denoted τ^+), investors would start playing the last phase of the previous equilibrium. Second, in the phase before news arrival, the trade-off between limit and market orders would no longer be affected by the likelihood of a news arrival event. Thus α_{eq} and l_{eq} would depend on neither μ nor ω.

Time priority. In the Online Appendix (Section I), I study the same model but with the time priority rule. Time priority creates new incentives to use limit orders. Indeed, a type-*ho* investor submits a limit order so that, if she becomes an *lo*, then she is better positioned in the queue. That advantage comes at a cost, however: her order might be executed while she is still of type *ho*, which would be suboptimal. The main problem with incorporating the time priority rule is that it makes the model considerably less tractable, even as the main economic forces at play continue to operate. Thus one can generate an equilibrium with identical strategies (e.g., *hn* and *lo* submit limit and market orders while *ho* and *ln* stay out) by making one of the following assumptions: (i) investors contact the market at a low enough frequency that the probability of an *ho* investor being able to cancel before execution remains sufficiently high; or (ii) the arrival of news is likely enough that an *ho* investor does not submit a limit order owing to the threat of its being (adversely) executed. I believe that extending the model along these lines would be a worthwhile contribution in its own right.

Asynchronous market contacts and asset value observation. In the model, an investor acquires up-to-date information about the asset each time he contacts the market. Outcomes would arguably differ if some agent actions could be taken without observing the asset value. In this case, the probability that "there was a news arrival since the last time the asset value was observed" increases with time. One would expect investors to cancel their limit orders when too much time has passed since they last observed the asset value.

Initial state of the order book. The Online Appendix (Section H) demonstrates how to treat the case where, at $t = 0$, the initial market depth is exogenous. In equilibrium, the market will converge to the initial steady state. Since news arrival will occur before convergence is complete, the market's depth at the start of the transition phase is no longer α_{eq}; however, it will be linked to α_{eq} because that value is the limit toward which the depth converges. Hence at each instant, the market depth shares the properties of α_{eq}.

Endogenous reaction delay. In the Online Appendix (section J), I consider a way to endogenize λ. That is, λ would be the choice of all investors in equilibrium. The first step is to chose an ex-ante objective function. I take the expected value function in which the probability of being *ho*, *hn*, *ln* or *lo* is the stationary probability distribution in the initial steady state. Then, I consider the problem of an investor who would unilaterally choose his reaction delay λ^*, at a cost $C(\lambda^*)$, while other investors' reaction delay is λ. I show that, if the cost function $C(.)$, is convex, the best response function $\lambda^*(\lambda)$ is the result of a well-defined concave optimization problem. Then, an endogenous λ would be such that $\lambda^*(\lambda) = \lambda$.

Multiplicity of Equilibria. The equilibrium described in Section 3 belongs to the class of equilibria described next. In this class, each equilibrium is characterized by three price pairs: (A, B), (A^u, B^u), and (A^d, B^d). Each bid-ask spread is equal to one tick, or Δ. The equilibrium strategy is qualitatively the same and differs from the symmetric equilibrium strategy only in the values taken by the equilibrium parameters. The only subset of this class of equilibria that exists for any value of μ is the one in which $B^u = B + \omega$, $A^u = A + \omega$, $B^d = B - \omega$, and $A^d = A - \omega$. These equilibria, indexed by (A, B), have exactly the same market depth α_{eq} as does the equilibrium described in Section 3.1. The main difference is that their execution rates for limit orders at prices A and B differ from each other, $l_{A,eq} \neq l_{B,eq}$. However, the sum of the two rates is the same across all equilibria: $l_{A,eq} + l_{B,eq} = 2l_{eq}$. In the model, these two execution rates affect the aggregate state of the book only through their sum. Hence it is fair to argue that studying the "symmetric" equilibrium is at least quasi-equivalent to studying the subclass of equilibria that exist for any value of μ.

There are also equilibria in which the order book is empty. In such an equilibrium, the *lo*- and *hn*-type investors coordinate on a trading price P at which to place (marketable) limit orders. The buy and sell order flows are exactly equal, which implies that their limit orders are immediately executed and that the limit order book is always empty. At $t = \tau$, the trading price adjusts immediately and there is no transition phase. One can rule out such equilibria since they are not "trembling hand" robust - and since a trembling-hand criterion can help filter out equilibria that exhibit too much coordination (Selten 1975).

6 Conclusion

The arrival of unscheduled news affects market dynamics. Following news arrival, investors become aware of news after a stochastic delay. An investor who reacts fast enough can benefit from an informational advantage. Because of this random delay, limit orders incur a relative adverse selection cost ex ante. When the frequency of news arrival increases, the adverse selection cost is greater and so market depth decreases. Hence prices adjust more quickly to news, and the trades' share of the orders' outflow - in the price adjustment process

- is greater. Moreover, there is a decrease in the ratio of (a) the trades' share of the orders' outflow during price adjustment to (b) the trades' share before news arrival. News arrival triggers a spike in trading volume, generates imbalances in order flows and in the order book, and is eventually followed by a change in price. Thus trading volume and price variability exhibit a positive covariance. Keeping the expected unconditional price variability constant, the covariance between trading volume and price variability declines with the news arrival frequency. Indeed, the frequency of news arrival has a negative effect on market depth before news arrival and therefore has a negative effect also on trading volume in the price adjustment process.

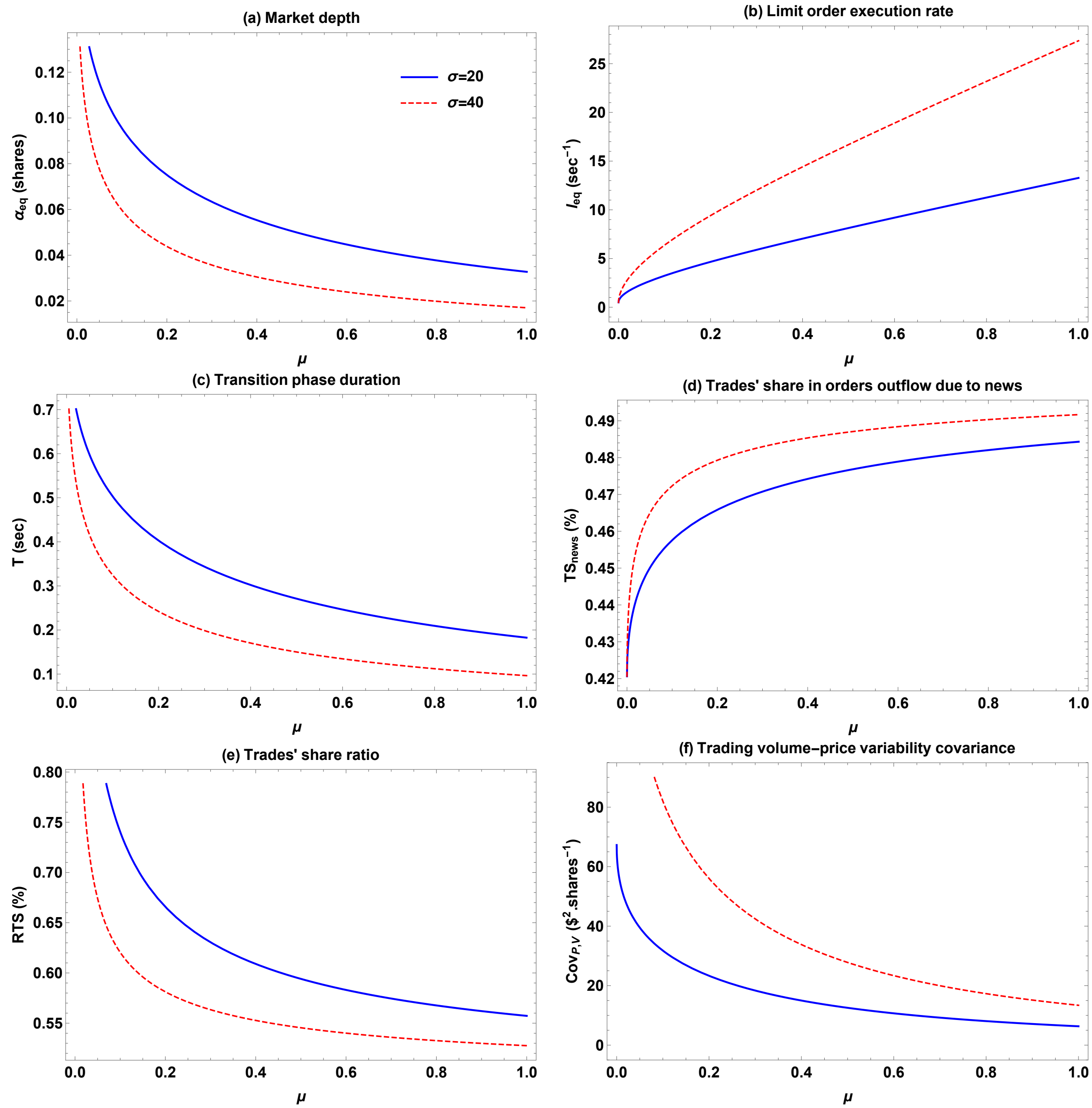

Figure 5: Effects of news arrival frequency. I analyze the effects of an increase of news arrival frequency, keeping unconditional price variability constant. The square root of price variability is set equal to $\sigma = 20$ (thick blue curves) or $\sigma = 40$ (red dashed curves). I set other parameters equal to $r = 1,\ \rho = 2,\ \Delta = 1,\ \delta = 6,\ \lambda = 10000$. The probability of news arrival increase from 0 to 0.5 as μ increases from 0 to 1. Panel (a) plots equilibrium market depth, α_{eq}. Panel (b) plots equilibrium limit order execution rate, l_{eq}. Panel (c) plots the transition duration T, expressed in seconds when the game horizon is a day (8 hours). Panel (d) plots the trades' share in orders' outflow due to news, TS_{news}. Panel (e) plots the ratio of trades' share, RTS. Panel (f) plots the trading volume-price variability covariance $Cov_{P,V}$.

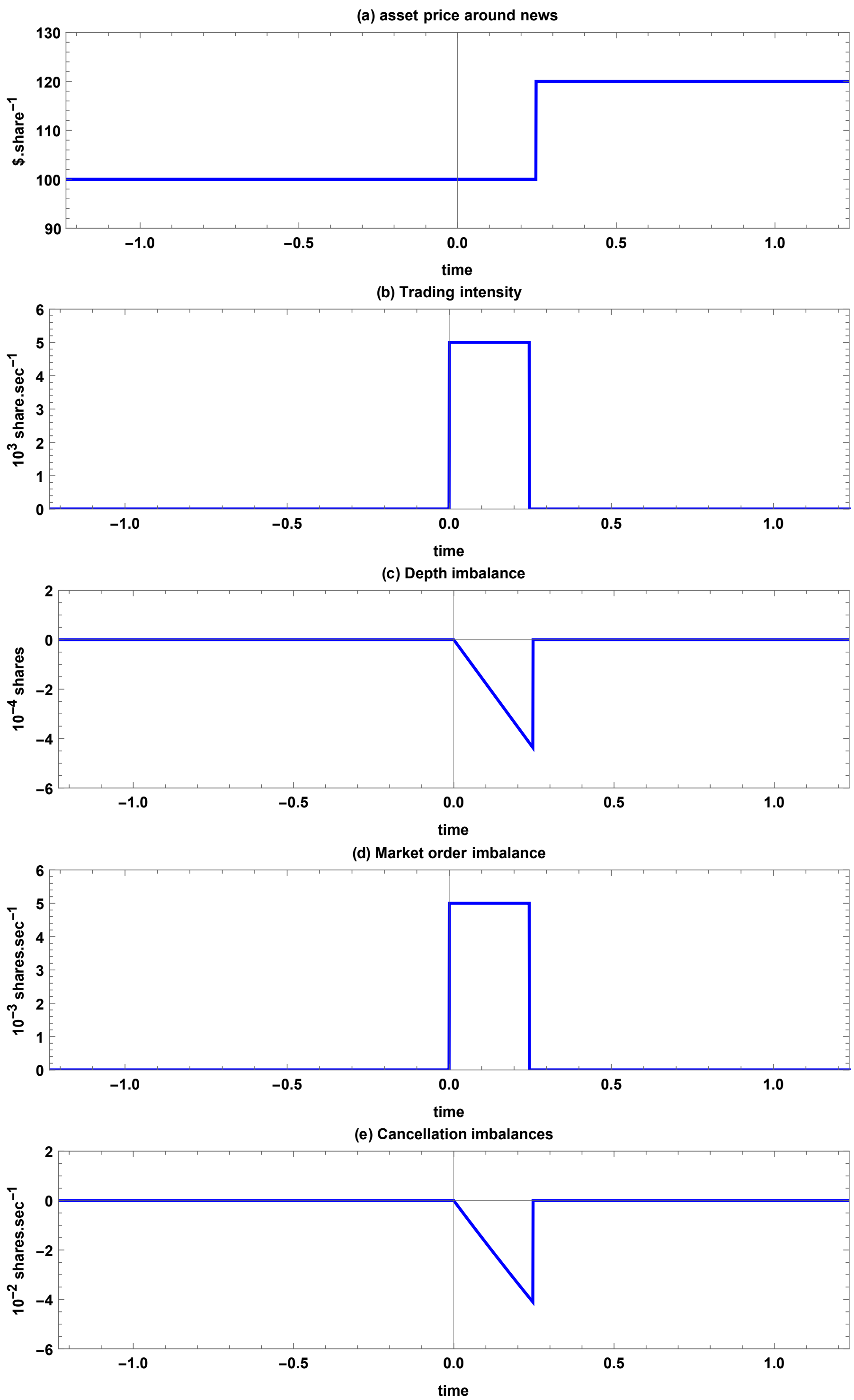

Figure 6: Market dynamics around news. In this figure, news is positive and occurs at $t = 0$. time is in seconds when the trading horizon is a trading day (8 hours). Panel (a) plots the asset price dynamics when the price change occurs at $t = 0.5$. Panel (b) plots the trading intensity. Panel (c) plots the depth imbalance between ask and bid sides, $DEimb_{A/B}(up)$. Panel (d) plots market order imbalance, $MOimb_{A/B}$. Panel (e) plots cancellation imbalance, $CAimb_{A/B}$. I set parameters equal to $\mu = 1,\ \omega = 20,\ r = 1,\ \rho = 2,\ \Delta = 1,\ \delta = 6,\ \lambda = 10000$.

A Proofs

A comprehensive version of each proposition's proof is given in the Online Appendix. That appendix also posits a fundamental lemma establishing that, if a strategy cannot be improved with a one-shot deviation, then no profitable deviation is possible. A *one-shot deviation from a strategy* is a strategy that deviates at the first (upcoming) contacting time yet thereafter follows the initial strategy.

A.1 Proof of proposition 1

The steady state is defined by the system

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} L_{ho} \\ L_{hn} \\ L_{lo} \\ L_{ln} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

First one can check that $L_{ho} = \frac{1}{2},\ L_{hn} = 0,\ L_{lo} = 0,\ L_{ln} = \frac{1}{2}$ is a particular solution of this system. Hence, the general space of solutions of this system is equal to

$$\begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} + \ker \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} + \text{vect} \left[\begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right]$$

It follows that there is an $\alpha \in \mathbb{R}$ such that $L_{ho} = L_{ln} = \frac{1}{2} - \alpha$, and $L_{lo} = L_{hn} = \alpha$.

A.2 Proof of propositions 3 and 4

type ho**.** An investor of type ho keeps his asset until he switches to the lo type:

$$(r + \rho)V_{ho} = \rho V_{lo} + rv$$

type ln**.** An investor of type ln does not place any order until she switches to the hn type:

$$(r + \rho)V_{ln} = \rho V_{hn}$$

type hn**.** An investor of type hn places a buy limit order, of which the probability of being executed is $l_B.dt$:

$$(r + \rho + l_B)V_{hn} = \rho V_{ln} + l_B(V_{ho} - B)$$

$$V_{hn} = V_{ho} - A$$

leading to

$$(r+\rho+l_B)(V_{ho}-A)=\rho V_{ln}+l_B(V_{ho}-B)$$

type ***lo*****.** An investor of type *lo* places a sell limit order, of which the probability of being executed is $l_A.dt$

$$(r+\rho+l_A)V_{lo}=rv-\delta+\rho V_{ho}+l_A(V_{ln}+A)$$

$$V_{lo}=V_{ln}+B$$

leading to

$$(r+\rho+l_A)(V_{ln}+B)=rv-\delta+\rho V_{ho}+l_A(V_{ln}+A)$$

One-shot deviation. To show that these are equilibrium strategies, I consider the value function generated by a one-shot deviation and verify that it is not profitable to deviate. (We show in Section A of the Online Appendix that this condition is both necessary and sufficient.) Take, for example, a *lo* type who stays out of the market until he becomes an *ho* type, at which point he starts to play the equilibrium strategy again. The corresponding value function is

$$(r+\rho)V_{lo-out}=rv-\delta+\rho V_{ho}$$

One can also rewrite the bellman equation for V_{lo} as

$$(r+\rho)V_{lo}=rv-\delta+\rho V_{ho}+l_A\Delta$$

Hence we clearly have

$$V_{lo}>V_{lo-out}$$

We verify that no other one-shot deviation is profitable in the online appendix (section B)

By replacing V_{hn} by $V_{ho}-A$ and V_{lo} by $V_{ln}+B$ it is easy to obtain that

$$(r+\rho)V_{ho}-\rho V_{ln}=rv+\rho B$$
$$(r+\rho)V_{ln}-\rho V_{ho}=-\rho A$$

and thus to the analogous expressions for V_{ho} and V_{ln}.

Replacing V_{ln} by $V_{lo}-B$ and V_{ho} by $V_{hn}+A$ in the equations of indifference between market and limit orders, one obtains

$$(r+\rho+l_B)(V_{ho}-A)=\rho(V_{lo}-B)+l_B(V_{ho}-B)$$
$$(r+\rho+l_A)(V_{ln}+B)=rv-\delta+\rho(V_{hn}+A)+l_A(V_{ln}+A)$$

which gives

$$rv + \rho B - (r+\rho)A = l_B(A-B)$$
$$-\rho A + (r+\rho)B - (rv-\delta) = l_A(A-B)$$

Since l_A and l_B must be positive numbers, it follows that

$$rv + \rho B - (r+\rho)A = rv - \rho\Delta - rA > 0, \text{ and } -\rho A + (r+\rho)B - (rv-\delta) = rB - (rv-\delta) - \rho\Delta > 0$$

which is verified by replacing A and B by their expressions in Assumption 2, since

$$rB - (rv-\delta) - \rho\Delta = rv - \rho\Delta - rA = \frac{\delta - (r+2\rho)\Delta}{2}$$

Finally,

$$V_{ho} = \frac{1}{r}\frac{1}{2}(rv - \rho\Delta) + \frac{1}{r+2\rho}\frac{1}{2}(rv + \rho(A+B)), \tag{54}$$
$$V_{ln} = \frac{1}{r}\frac{1}{2}(rv - \rho\Delta) - \frac{1}{r+2\rho}\frac{1}{2}(rv + \rho(A+B)), \tag{55}$$
$$V_{hn} = V_{ho} - A, \tag{56}$$
$$V_{lo} = V_{ln} + B. \tag{57}$$

Since,

$$v + \rho(A+B) = rv + \rho\left(2v - \frac{\delta}{r}\right) = (r+2\rho)v - \frac{\delta}{r}$$

It follows that

$$V_{ho} = v - \frac{\rho}{2r(r+2\rho)}(\delta + (r+2\rho)\Delta), \tag{58}$$
$$V_{ln} = \frac{\rho}{2r(r+2\rho)}(\delta - (r+2\rho)\Delta), \tag{59}$$
$$V_{hn} = -\frac{\rho\delta}{2r(r+2\rho)} - \frac{\rho\Delta}{2r} + \frac{\delta}{2r} - \frac{\Delta}{2} = \frac{r+\rho}{2r(r+2\rho)}(\delta - (r+2\rho)\Delta), \tag{60}$$
$$V_{lo} = \frac{\rho\delta}{2r(r+2\rho)} - \frac{\rho\Delta}{2r} + v - \frac{\delta}{2r} - \frac{\Delta}{2} = v - \frac{r+\rho}{2r(r+2\rho)}(\delta + (r+2\rho)\Delta). \tag{61}$$

A.3 Proof of proposition 6

Consider the case where the new asset value is $v+\omega$. We can start by writing the value function of a lo-type investor who has already reacted to news and has resubmitted her limit order at price A^u. This investor, $lo-A^u$, keeps her order in the book until she becomes of type ho or until the transition phase ends. Therefore,

$$(r+\rho)V^u_{lo-A^u}(t) = r(v+\omega) - \delta + \frac{\partial V^u_{lo-A^u}}{\partial t} + \rho V^u_{ho}(t). \tag{62}$$

By continuity, at the end of the transition phase (i.e., at time $\tau + T$) the value function of this investor is her value function in the last phase of the game, which is also her value function in the asymptotic steady state defined in Proposition 4. Hence one may write

$$V^u_{lo-A^u}(\tau + T) = v + \omega - \frac{r+\rho}{2r(r+2\rho)}(\delta + (r+2\rho)\Delta) \tag{63}$$

One can similarly define the value function of other types. Regardless of whether they contact the market, the ho types stay out. They remain out until becoming $lo - A^u$ types or until the transition phase ends. Thus,

$$(r+\rho)V^u_{ho}(t) = r(v+\omega) + \frac{\partial V^u_{ho}}{\partial t} + \rho V^u_{lo-A^u}(t), \tag{64}$$

$$\text{with } \; V^u_{ho}(\tau + T) = v + \omega - \frac{\rho}{2r(r+2\rho)}(\delta + (r+2\rho)\Delta). \tag{65}$$

As soon as they contact the market, type-ln investors send their respective buy market orders and immediately behave as their new type (lo or ho); then

$$(r+\rho+\lambda)V^u_{ln}(t) = \frac{\partial V^u_{ln}}{\partial t} + \rho(V^u_{ho}(t) - A) + \lambda(V^u_{lo-A^u}(t) - A), \tag{66}$$

$$\text{with } \; V^u_{ln}(\tau + T) = \frac{\rho}{2r(r+2\rho)}(\delta - (r+2\rho)\Delta). \tag{67}$$

As soon as the hn types contact the market, they send their buy market orders and immediately behave as their new type (ho or lo); then

$$(r+\rho+\lambda)V^u_{hn}(t) = \frac{\partial V^u_{hn}}{\partial t} + \rho(V^u_{lo-A^u}(t) - A) + \lambda(V^u_{ho}(t) - A), \tag{68}$$

$$\text{and } \; V^u_{hn}(\tau + T) = \frac{r+\rho}{2r(r+2\rho)}(\delta - (r+2\rho)\Delta). \tag{69}$$

Finally, one can define the value function of a lo type with a limit order at price A. As soon as they contact the market, these $lo - A$ types cancel their limit orders and become either type $lo - A^u$ or type ho. However, if such an investor's order were executed in the meantime, then in this case he would become a type ln and receive the payment A. The execution rate of limit order is equal to $k(t)$, the flow of market orders sent by types hn and ln divided by market depth, $D(t)$:

$$k(t) = \frac{\lambda + \rho}{2D(t)} \tag{70}$$

One can then define the value function

$$(r+\rho+\lambda+k(t))V^u_{lo-A}(t) = r(v+\omega) - \delta + \frac{\partial V^u_{lo-A}}{\partial t} + \rho V^u_{ho}(t) + \lambda V^u_{lo-A^u}(t) + k(t)(V^u_{ln}(t) + A) \tag{71}$$

At the end of the transition phase, if the investor still has a limit order at price A then that order is executed with certainty. Therefore,

$$V^u_{lo-A}(\tau + T) = V^u_{ln}(\tau + T) + A = v - \frac{r+\rho}{2r(r+2\rho)}(\delta + (r+2\rho)\Delta) + \Delta \tag{72}$$

Now we must check that there is no profitable one-shot deviation for an investor of type lo.

- Instead of staying at A^u an lo type could place a sell market order at price B and obtain $V^u_{ln}(t) + B$. This is clearly not a profitable strategy given that type-ln investors immediately buy at price A.

- Another deviation could be to keep the limit order at price A; the corresponding value function is $V^u_{lo-A}(t)$. After putting $X(t) = V^u_{lo-A^u}(t) - V^u_{lo-A}(t)$ one obtain the ODE

$$(r + \rho + \lambda + k(t))X(t) = \frac{\partial X}{\partial t} + k(t)(V^u_{lo-A^u}(t) - A - V^u_{ln}(t))$$

  The solution of this ODE is of the form,

$$X(t) = e^{\int_0^t (r+\rho+\lambda+k(s))ds}[C - \int_0^t k(s)(V^u_{lo-A^u}(s) - V^u_{ln}(s) - A)e^{-\int_0^s (r+\rho+\lambda+k(l))dl} ds],$$

  where C is a constant. Then, $X(t) \times e^{-\int_0^t (r+\rho+\lambda+k(s))ds}$ is decreasing. Indeed, (in the Online Appendix, Section D.2, p.11) it is shown that it is optimal for an investor of type ln to send a buy market order, that is, for all $t \in [\tau, \tau + T)$, $V^u_{lo-A^u}(t) - A - V^u_{ln}(t) \geq 0$. Moreover, $X(\tau + T) = \bar{V}^u_{lo-A^u} - A - \bar{V}^u_{ln-out} = B^u - A > 0$. Therefore, $X(t)$ is positive for all $t \in [0, T]$. This deviation is not profitable.

- A lo type could send a limit order at any price $A < A' < A^u$. The corresponding value function, $V(t)$, would be defined as

$$\begin{aligned}(r + \rho + \lambda)V(t) &= r(v + \omega) - \delta + \frac{\partial V}{\partial t} + \rho V^u_{ho-out}(t) + \lambda V^u_{lo-A^u}(t) \\ &= \frac{\partial V}{\partial t} + (r + \rho + \lambda)V^u_{lo-A^u}(t) - \frac{\partial V^u_{lo-A^u}}{\partial t}\end{aligned}$$

  That solution, $V(t)$, to this differential equation is such that

$$V^u_{lo-A^u}(t) - V(t) = C \times e^{(r+\rho+\lambda)t}$$

  where C is a constant. The order would be executed at $\tau + T$. Therefore, $V^u_{lo-A^u}(\tau + T) - V(\tau + T) = \bar{V}^u_{lo-A^u} - A' - \bar{V}^u_{ln} = B^u - A' > 0$. The deviation is not profitable.

- Finally, an lo type could stay out. The corresponding value function, $V(t)$, would be defined as

$$\begin{aligned}(r + \rho + \lambda)V(t) &= r(v + \omega) - \delta + \frac{\partial V}{\partial t} + \rho V^u_{ho-out}(t) + \lambda V^u_{lo-A^u}(t) \\ &= \frac{\partial V}{\partial t} + (r + \rho + \lambda)V^u_{lo-A^u}(t) - \frac{\partial V^u_{lo-A^u}}{\partial t}\end{aligned}$$

  which again would give

$$V^u_{lo-A^u}(t) - V(t) = C \times e^{(r+\rho+\lambda)t}.$$

  Then, at T we would have, $V^u_{lo-A^u}(\tau + T) - V(\tau + T) = \bar{V}^u_{lo-A^u} - \bar{V}^u_{lo-out} > 0$.

The proofs for other types - and for the case where the new asset value is $v - \omega$ - are similar

and are given in Section D of the Online Appendix.

A.4 Proof of proposition 7

At $t = \tau$ we have

$$D_A = L_{lo} = \alpha, \quad D_B = L_{hn} = \alpha$$

Suppose the new asset value is $v + \omega$, and let $t > \tau$. Then the dynamics of market depth at price A are driven by type-lo investors, with limit orders at A, canceling their limit orders or seeing them executed via market orders sent by hn and ln. Hence we can write

$$\frac{\partial D_A}{\partial t} = -(\lambda + \rho) D_A(t) - (\lambda + \rho) L_{hn}(t) - (\lambda + \rho) L_{ln}(t)$$

since $L_{ln}(t) + L_{hn}(t) = \frac{1}{2}$, it is equivalent to

$$\frac{\partial D_A}{\partial t} = -(\lambda + \rho) D_A(t) - \frac{1}{2}(\lambda + \rho)$$

The solution of this ODE is,

$$D_A(t) = D(t) = -\frac{1}{2} + \left(\alpha + \frac{1}{2}\right) e^{-(\rho+\lambda)(t-\tau)}$$

The transition phases ends at $T + \tau$ such that $D(T + \tau) = 0$, that is,

$$T = \frac{1}{\rho + \lambda} \ln(1 + 2\alpha)$$

A.5 Proof of proposition 8

To identify $l(\alpha)$, the value at which investors are indifferent between limit and market orders, we must perform the following calculations:

$$\begin{aligned} V^u_{ln}(\tau) + A - V^u_{lo-A}(\tau) = V^d_{ho}(\tau) - B - V^d_{hn-B}(\tau) &= -[r\omega - \frac{\delta}{2} - r\frac{\Delta}{2}] \int_\tau^{\tau+T} \frac{D(t)e^{-r(t-\tau)}}{\alpha} d \\ &= -[r\omega - \frac{\delta}{2} - r\frac{\Delta}{2}] \int_\tau^{\tau+T} h(t) e^{-(r+\rho+\lambda)(t-\tau)} dt \\ V^d_{ln}(\tau) + B - V^d_{lo-A}(\tau) = V^u_{ho}(\tau) - A - V^u_{hn-B}(\tau) &= \frac{r\omega + \frac{\delta}{2} - r\frac{\Delta}{2}}{r + \rho + \lambda} + \frac{(\lambda + \rho)\omega - \rho\Delta}{r + \rho + \lambda} e^{-(r+\rho+\lambda)T} \end{aligned}$$

with $D(t)$ given in proposition 7 and $h(t) = \frac{D(t)}{\alpha e^{-(\lambda+\rho)(t-\tau)}} = 1 - \frac{1}{2}\frac{(1-e^{-(\lambda+\rho)(t-\tau)})}{\alpha e^{-(\lambda+\rho)(t-\tau)}} = 1 + \frac{1}{2\alpha} - \frac{1}{2\alpha}e^{(\lambda+\rho)(t-\tau)}$ and $T = \frac{\ln(1+2\alpha)}{\rho+\lambda}$, noting that $h(\tau+T) = 0$.

$$l(\alpha) = \frac{\delta - (r+2\rho)\Delta}{2\Delta} + \frac{1}{2\Delta}\mu\left[-\Delta - (r\omega - \frac{\delta}{2} - r\frac{\Delta}{2})\int_{\tau}^{\tau+T} h(t)e^{-(r+\rho+\lambda)(t-\tau)}dt\right]$$
$$+ \frac{1}{2\Delta}\mu\left[\frac{r\omega + \frac{\delta}{2} - r\frac{\Delta}{2}}{r+\rho+\lambda} + \frac{(\lambda+\rho)\omega - \rho\Delta}{r+\rho+\lambda}e^{-(r+\rho+\lambda)T}\right]$$

The derivative of $\int_{\tau}^{\tau+T} h(t)e^{-(r+\rho+\lambda)(t-\tau)}dt$ with respect to α is equal to

$$\frac{\partial T}{\partial \alpha}h(\tau+T)e^{-(r+\rho+\lambda)T} + \int_{\tau}^{\tau+T}\frac{\partial h}{\partial \alpha}(t)e^{-(r+\rho+\lambda)(t-\tau)}dt = \int_{\tau}^{\tau+T}\frac{\partial h}{\partial \alpha}(t)e^{-(r+\rho+\lambda)(t-\tau)}dt > 0$$

It implies that $\frac{\partial l}{\partial \alpha} < 0$. Our aim in what follows is to show that, on $[0, 1/4]$, there is a unique solution to equation (38), that is equivalently to the following equation,

$$G(\alpha) = \alpha \times (2l(\alpha) + 2\rho) = \frac{\rho}{2}$$

Analysis of the function G (detailed version in the online appendix, section E)

$$G(\alpha)\Delta = (\delta - (r+\mu)\Delta)\alpha - \mu(r\omega - \frac{\delta}{2} - r\frac{\Delta}{2})\int_{\tau}^{\tau+T} D(t)e^{-r(t-\tau)}dt$$
$$+ \mu\alpha\left[\frac{r\omega + \frac{\delta}{2} - r\frac{\Delta}{2}}{r+\rho+\lambda} + \frac{(\lambda+\rho)\omega - \rho\Delta}{r+\rho+\lambda}e^{-(r+\rho+\lambda)T}\right]$$
$$= (\delta - (r+\mu)\Delta)\alpha - \mu(r\omega - \frac{\delta}{2} - r\frac{\Delta}{2})\int_{\tau}^{\tau+T} D(t)e^{-r(t-\tau)}dt$$
$$+ \mu\alpha\left[\frac{r\omega + \frac{\delta}{2} - r\frac{\Delta}{2}}{r+\rho+\lambda}(1 - e^{-(r+\rho+\lambda)T}) + \left(\frac{r\omega + \frac{\delta}{2} - r\frac{\Delta}{2}}{r+\rho+\lambda} + \frac{(\lambda+\rho)\omega - \rho\Delta}{r+\rho+\lambda}\right)e^{-(r+\rho+\lambda)T}\right]$$
$$= (\delta - (r+\mu)\Delta)\alpha - \mu(r\omega - \frac{\delta}{2} - r\frac{\Delta}{2})\int_{\tau}^{\tau+T} D(t)e^{-r(t-\tau)}dt$$
$$+ \mu\left[\left(r\omega + \frac{\delta}{2} - r\frac{\Delta}{2}\right)\int_{\tau}^{\tau+T}\alpha e^{-(r+\rho+\lambda)(t-\tau)}dt + \alpha\left(\omega + \frac{\frac{\delta - r\Delta}{2} - \rho\Delta}{r+\rho+\lambda}\right)e^{-(r+\rho+\lambda)T}\right]$$

Remind that $D(t) = -\frac{1}{2} + \left(\alpha + \frac{1}{2}\right)e^{-(\rho+\lambda)(t-\tau)}$ then

$$G(\alpha)\Delta = (\delta - (r+\mu)\Delta)\alpha - \mu(r\omega - \frac{\delta}{2} - r\frac{\Delta}{2})\int_{0}^{T}\left(-\frac{1 - e^{-(\rho+\lambda)t}}{2}e^{-rt} + \alpha e^{-(r+\rho+\lambda)t}\right)dt$$
$$+ \mu\left[\left(r\omega + \frac{\delta}{2} - r\frac{\Delta}{2}\right)\int_{0}^{T}\alpha e^{-(r+\rho+\lambda)t}dt + \alpha\left(\omega + \frac{\frac{\delta - r\Delta}{2} - \rho\Delta}{r+\rho+\lambda}\right)e^{-(r+\rho+\lambda)T}\right]$$
$$= (\delta - r\Delta)\alpha + \mu(\omega - \frac{\delta}{2} - r\frac{\Delta}{2})\int_{0}^{T}\frac{1 - e^{-(\rho+\lambda)t}}{2}e^{-rt}dt + \mu\alpha\delta\int_{0}^{T}e^{-(r+\rho+\lambda)t}dt$$
$$+ \mu\alpha\left[\left(\omega + \frac{\frac{\delta - r\Delta}{2} - \rho\Delta}{r+\rho+\lambda}\right)e^{-(r+\rho+\lambda)T} - \Delta\right]$$

First, I can ensure that $G(1/4)\Delta - \rho\Delta/2 > 0$,

$$G(1/4)\Delta - \frac{\rho\Delta}{2} = \frac{\delta - (r+2\rho)\Delta}{4} + \mu(r\omega - \frac{\delta}{2} - r\frac{\Delta}{2})\int_0^{T(1/4)} \frac{1-e^{-(\rho+\lambda)t}}{2}e^{-rt} + \frac{\mu}{4}\delta\int_0^{T(1/4)} e^{-(r+\rho+\lambda)t}$$
$$+\frac{\mu}{4}\left[\left(\omega + \frac{\frac{\delta - r\Delta}{2} - \rho\Delta}{r+\rho+\lambda}\right)\frac{1}{(3/2)^{\frac{r}{\rho+\lambda}+1}} - \Delta\right]$$

In order to verify this expression for all values of μ and λ, two conditions are sufficient:

$$\omega > \frac{\delta + r\Delta}{2r}, \text{ and } \omega > \Delta(3/2)^{\frac{r}{\rho+\lambda}+1}$$

Let's define

$$g(\alpha) = \alpha\left[\left(\omega + \frac{\frac{\delta - r\Delta}{2} - \rho\Delta}{r+\rho+\lambda}\right)\frac{1}{(1+2\alpha)^{\frac{r}{\rho+\lambda}+1}} - \Delta\right]$$

Since, under the previous conditions, T increases with α, we know that - for any μ and λ - if g increases with α then also G increases with α:

$$g'(\alpha) = \left(\omega + \frac{\frac{\delta - r\Delta}{2} - \rho\Delta}{r+\rho+\lambda}\right)\frac{1}{(1+2\alpha)^{\frac{r}{\rho+\lambda}+1}} - \Delta - \left(\frac{r}{\rho+\lambda}+1\right)\left(\omega + \frac{\frac{\delta - r\Delta}{2} - \rho\Delta}{r+\rho+\lambda}\right)\frac{2\alpha}{(1+2\alpha)^{\frac{r}{\rho+\lambda}+2}}$$
$$= \left(1 - \left(\frac{r}{\rho+\lambda}+1\right)\frac{2\alpha}{1+2\alpha}\right)\left(\omega + \frac{\frac{\delta - r\Delta}{2} - \rho\Delta}{r+\rho+\lambda}\right)\frac{1}{(1+2\alpha)^{\frac{r}{\rho+\lambda}+1}} - \Delta$$

A sufficient condition is

$$\omega > \frac{\Delta(3/2)^{\frac{r}{\rho+\lambda}+1}}{1 - \frac{1}{3}\left(\frac{r}{\rho+\lambda}+1\right)}$$

Other equilibrium condition. The other equilibrium condition is

$$Y = V_{hn} - V_{hn-B^d} = V_{lo} - V_{lo-A^u} > 0$$

I show in the online appendix (section F) that

$$(r+\rho+\mu)Y = \frac{\delta - (r+2\rho)\Delta}{2} - \frac{\mu}{2}\Delta\left(1 - e^{-(r+\rho+\lambda)T}\right) + \frac{\mu}{2}\left[\frac{\delta}{r+\rho+\lambda} - \frac{\delta + (r+2\rho)\Delta}{2(r+\rho+\lambda)}e^{-(r+\rho+\lambda)T}\right]$$

A sufficient condition (see online appendix, section F) for Y to be positive is

$$\frac{4}{9}\omega - \Delta > \Delta\frac{2\rho\Delta}{\delta - (r+2\rho)\Delta}$$

Threshold value. From the various preceding elements of the proof, it follows that $\hat{\omega}(\delta, \Delta)$ exists and has an upper bound:

$$\widehat{\omega}(\delta, \Delta) < \max\left[\frac{\delta + r\Delta}{2r}, \frac{\Delta(3/2)^{\frac{r}{\rho+\lambda}+1}}{1 - \frac{1}{3}\left(\frac{r}{\rho+\lambda}+1\right)}, \frac{2\Delta}{9}\left(1 + \frac{2\rho\Delta}{\delta - (r+2\rho)\Delta}\right)\right]$$

A.6 Proof of proposition 9

$$G(\alpha) = \alpha \times (2l(\alpha) + 2\rho) = \frac{\rho}{2} \Leftrightarrow \alpha \left[\delta - r\Delta + \mu\left(\frac{\omega}{1+2\alpha} - \Delta\right)\right] = \frac{\rho\Delta}{2}$$

Which is equivalent to

$$U(\alpha) = 2[\delta - (r+\mu)\Delta]\alpha^2 + [\delta - (r+\rho)\Delta + \mu(\omega - \Delta)]\alpha = \frac{\rho\Delta}{2}$$

First $\delta - (r+\rho)\Delta + \mu(\omega-\Delta)$ is always positive. If $\delta - (r+\mu)\Delta > 0$ then $U(\alpha)$ is positive and increasing for $\alpha > 0$. The solution of the equation is the largest root of $U(\alpha) - \rho\Delta/2$, the other one being negative. The expression of this root is

$$\frac{-[\delta - (r+\rho)\Delta + \mu(\omega-\Delta)] + \sqrt{[\delta - (r+\rho)\Delta + \mu(\omega-\Delta)]^2 + 4\rho\Delta[\delta - (r+\mu)\Delta]}}{4[\delta - (r+\mu)\Delta]}$$

If $\delta - (r+\mu)\Delta < 0$ then $U(\alpha)$ has a root equals to 0 and a positive one. Hence $U(\alpha) - \rho\Delta/2$ has two positive roots (or none). We know that $U(1/4) - \rho\Delta/2 > 0$. α_{eq} must be lower than 1/4, then it is the smallest roots of $U(\alpha) - \rho\Delta/2$. The expression of this root is

$$\frac{[\delta - (r+\rho)\Delta + \mu(\omega-\Delta)] - \sqrt{[\delta - (r+\rho)\Delta + \mu(\omega-\Delta)]^2 - 4\rho\Delta[(r+\mu)\Delta - \delta]}}{4[(r+\mu)\Delta - \delta]}$$

Overall, we have

$$\alpha_{eq} = \begin{cases} \frac{\delta-(r+\rho)\Delta+\mu(\omega-\Delta)}{4(\delta-(r+\mu)\Delta)}\left(\sqrt{1 + 4\rho\Delta\frac{\delta-(r+\mu)\Delta}{(\delta-(r+\rho)\Delta+\mu(\omega-\Delta))^2}} - 1\right) & \text{if } \mu < \frac{\delta}{\Delta} - r \\ \frac{\delta-(r+\rho)\Delta+\mu(\omega-\Delta)}{4((r+\mu)\Delta-\delta)}\left(1 - \sqrt{1 - 4\rho\Delta\frac{(r+\mu)\Delta-\delta}{(\delta-(r+\rho)\Delta+\mu(\omega-\Delta))^2}}\right) & \text{if } \mu > \frac{\delta}{\Delta} - r \end{cases} \quad (73)$$

A.7 Proof of proposition 10

Here I consider the case where $\lambda = \infty$, but the proof is similar in the general case. We know that in $\alpha = \alpha_{eq}$, G is increasing. Note that,

$$G(\alpha) = \alpha\left[\delta - r\Delta + \mu\left(\frac{\omega}{r}\frac{1}{1+2\alpha} - \Delta\right)\right]$$

Hence, G is clearly increasing with μ and ω, which leads to the fact that α_{eq} decreases with μ and ω. Next I consider the case where the unconditional price variability is constant, that is $\frac{\mu\omega^2}{r+\mu} = \sigma^2 \Leftrightarrow \omega = \sigma\sqrt{1+\frac{r}{\mu}}$, with $\sigma > \widehat{\omega}(\delta, \Delta)$. In this case we can rewrite G,

$$G(\alpha) = \alpha\left[\delta - r\Delta + \mu\left(\sqrt{1+\frac{r}{\mu}}\frac{\sigma}{r}\frac{1}{1+2\alpha} - \Delta\right)\right]$$

$$\frac{\partial G}{\partial \mu} = \alpha \left(\sqrt{1+\frac{r}{\mu}} \frac{\sigma}{r} \frac{1}{1+2\alpha} - \Delta - \mu \frac{r}{\mu^2} \frac{1}{2\sqrt{1+\frac{r}{\mu}}} \frac{\sigma}{r} \frac{1}{1+2\alpha} \right) = \alpha \left(\frac{2+\frac{r}{\mu}}{2\sqrt{1+\frac{r}{\mu}}} \frac{\sigma}{r} \frac{1}{1+2\alpha} - \Delta \right)$$

$$= \alpha \left(\frac{\sigma}{2r} \frac{1}{1+2\alpha} \left(\sqrt{1+\frac{r}{\mu}} + \frac{1}{\sqrt{1+\frac{r}{\mu}}} \right) - \Delta \right) > \alpha \left(\frac{\sigma}{r} \frac{1}{1+2\alpha} - \Delta \right) > 0$$

Hence, when the unconditional price variability is fixed, α_{eq} decreases with μ and increases with ω.

A.8 Proof of proposition 11

The amount of trade is given by

$$\int_0^T \frac{\rho+\lambda}{2} dt = \frac{\rho+\lambda}{2} T = \frac{1}{2} \ln(1+2\alpha_{eq})$$

A.9 Proof of corollary 1

The derivative of function $f(x) = \ln(1+x)/x$ with respect to x is equal to

$$f'(x) = \frac{x-(1+x)\ln(1+x)}{(1+x)x^2},$$

which is equal to 0 for $x = 0$. The derivative of the numerator is equal to $-\ln(1+x) < 0$. Hence, $f' < 0$ for $x > 0$. f decreases with x, and $1/f$ increases.

$$TS_{news} = \frac{\ln(1+2\alpha_{eq})}{2\alpha_{eq}}$$

When α_{eq} decreases, due to higher news arrival, TS_{news} increases.

A.10 Proof of proposition 12

The derivative of $\frac{\ln(1+2x)}{4x(1-4x)}$ with respect to x is equal to

$$\frac{2}{(1+2x)4x(1-4x)} - \frac{(4-32x)\ln(1+2x)}{(4x(1-4x))^2} = \frac{2x(1-4x)-(1-8x)(1+2x)\ln(1+2x)}{4x^2(1+2x)(1-4x)^2}$$

$$\geq \frac{2x[1-4x-(1-8x)(1+2x)]}{4x^2(1+2x)(1-4x)^2} \quad \text{(because } \ln(1+2x) \leq 2x\text{)}$$

$$\geq \frac{2x(2x+16x^2)}{4x^2(1+2x)(1-4x)^2} > 0 \quad \text{(for } x > 0\text{)}$$

References

[1] Biais B., Hombert J. and Weill P.-O. (2014), "Equilibrium Pricing and Trading Volume under Preference Uncertainty", *Review of Economic Studies* 81, 1401-1437.

[2] Cartea A., Donnelly R., and Jaimungal S. (2015), "Enhancing Trading Strategies with Order Book Signals", *Working paper.*

[3] Cont R., Kukanov A. and Stoikov S. (2014), "The Price Impact of Order Book Events", *Journal of Financial Econometrics* 12, 47-88.

[4] Della Vigna S. and Pollet J. M. (2009), "Investor inattention and Friday earnings announcements", *Journal of Finance* 64, 709-749.

[5] Duffie D., Gârleanu N. and Pedersen L. (2005), "Over-the-Counter Markets", *Econometrica* 73, 1815-1847.

[6] Duffie D., Gârleanu N. and Pedersen L. (2007), "Valuation in over-the-counter markets", *Review of Financial Studies* 20, 1865-1900.

[7] Ederington L.H., and Lee J. (1995), "The short-run dynamics of the price adjustment to new information", *Journal of Financial and Quantitative Analysis* 30, 117-134.

[8] Fleming M. J. and Remolona E. M. (1999), "Price formation and liquidity in the U.S. treasury market: the response to public information", *Journal of Finance* 54, 1901-1915.

[9] Foucault T. (1999), "Order flow composition and trading costs in a dynamic limit order market", *Journal of Financial Markets* 2, 99-134.

[10] Foucault T., Kadan O. and Kandel E. (2005), "Limit order book as a market for liquidity", *Review of Financial Studies* 18, 1171-1217.

[11] Gallant A.R., Rossi P.E. and Tauchen G. (1992), "Stock Prices and Volume", *Review of Financial Studies* 5, 199-242.

[12] Glosten L. and Milgrom P.(1985), "Bid, ask and transaction prices in specialist market with heterogeneously informed traders", *Journal of Financial Economics* 13, 71-100.

[13] Goettler R., Parlour C. and Rajan U. (2005), "Equilibrium in a dynamic limit order market", *Journal of Finance* 60, 2149-2192.

[14] Goettler R., Parlour C. and Rajan U. (2009), "Informed traders and limit order markets", *Journal of Financial Economics* 93, 67-87.

[15] Graham J.R., Koski J. L and Loewenstein U. (2006), "Information Flow and Liquidity around Anticipated and Unanticipated Dividend Announcements", *Journal of Business* 79, 2301-2336

[16] Green T. C. (2004), "Economic news and the impact of trading on bond prices", *Journal of Finance* 59, 1201-1233

[17] Gross-Klussmann A. and Hautsch N. (2011), "When machines read the news: Using automated text analytics to quantify high frequency news-implied market reactions", *Journal of Empirical Finance* 18, 321-340.

[18] Hendershott T. and Moulton P.C. (2011), "Automation, speed,and stock market quality: The NYSE's Hybrid", *Journal of Financial Markets* 14, 568-604

[19] Karpoff J. (1987), "The Relation between Price Changes and Trading Volume: A Survey", *Journal of Financial and Quantitative Analysis* 22, 109-126.

[20] Kwan A. and Philip R. (2015), "High-frequency trading and execution costs", *Working paper.*

[21] Kyle A. (1985), "Continuous auctions and insider trading", *Econometrica* 53, 1315-1336.

[22] Lagos R. and Rocheteau G. (2009), "Liquidity in asset markets with search frictions", *Econometrica* 77, 403-426.

[23] Lagos R., Rocheteau G. and Weill P.-O. (2011), "Crises and liquidity in over-the-counter markets", *Journal of Economic Theory* 146, 2169-2205.

[24] O'Hara M, Saar G. and Zhong Z. (2015), "Relative Tick Size and the Trading Environment", *Working paper.*

[25] Pagnotta E. and Philippon T. (2015), "Competing on speed", *Working Paper.*

[26] Parlour C. (1998), "Price dynamics in a limit order market", *Review of Financial Studies* 11, 789-816.

[27] Riordan R. and Storkenmaier A. (2012), "Latency, liquidity and price discovery", *Journal of Financial Markets* 15, 416-437.

[28] Rosu I. (2009), "A dynamic model of the limit order book", *Review of Financial Studies* 22, 4601-4641.

[29] Rosu I. (2015), "Liquidity and information in order driven markets", *Working Paper.*

[30] Selten, R. (1975), "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games", *International Journal of Game Theory* 4, 25-55

[31] Tauchen G. E. and Pitt M. (1983), "The Price Variability-Volume Relationship on Speculative Markets", *Econometrica* 51, 485-504.

[32] Vayanos D. and Weill P.-O. (2008), "A search-based theory of the on-the-run phenomenon", *Journal of Finance* 63, 1351-1389

[33] Weill P.-O. (2007), "Leaning against the wind", *Review of Economic Studies* 74, 1329-1354.

[34] Weill P.-O. (2008), "Liquidity premia in dynamic bargaining markets", *Journal of Economic Theory* 140, 66-96.

[35] Yueshen B. Z. (2014), "Queuing Uncertainty", *Working paper.*