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# The Effects of Environmental Quality Misperception on Investments and Regulation*

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**Abstract**

In this paper we analyse a setup where consumers are heterogeneous in the perception of environmental quality. The equilibrium is verified in a setting with horizontal and vertical (green) differentiation. Profits are increasing in the misperception of quality, while the investment in green quality decreases the more the goods are substitutes. We further consider the introduction of either an emission tax or an environmental standard. Both interventions rise the investment in quality, but their effect is differently influenced by the level of environmental misperception. We show that an optimal environmental standard may be effective against greenwashing when the marginal damage of emissions is low.

**JEL codes:** L13, L51, Q50.

**Keywords:** Green quality; Misperception; Pigouvian taxation; Environmental Standard.

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# 1 Introduction

Access to knowledge and innovations in technology have led to increasing awareness of environmental issues. Several studies have shown that world-wide, consumer's appetite for green products has increased significantly in the past years (Chase and Smith, 1992; Kim and Choi, 2005; Chen, 2008, *inter alia*). Communities demand cleaner environments and the rise of consciousness has resulted in significant environmental improvement (Reitman, 1992). More companies around the world have reacted by developing eco-friendly products (Kohl, 1991; Chang, 2011).

In reality, there may be a perception in the marketplace that a green package equals greater value. With little knowledge to make an informed decision, individuals seem to rely on firms' packaging to understand the positive (or negative) impact of production. This should not come as too much of a surprise, as consumers have traditionally lacked a detailed understanding of green innovation. Some consumers would undertake larger green consumption if they better understood the impact that each production has on the society. Others would confuse the level of transparency when there are no specific criteria qualifying a product as green.[1]

The marketing strategy to appear environmentally friendly is called "greenwashing". It usually occurs when a company spends more time and money claiming green production through advertising and marketing than actually implementing business practices that minimize environmental impact.[2] Firms may operate in a way that is damaging to the environment or in an opposite manner to the goal of the announced initiatives.[3] Since consumers' evaluation is based on advertising to make decisions, greenwashing distorts their confidence (Hamann and Kapelus, 2004) and therefore it is positively associated to confusion and perceived risk (Chen and Chang, 2013). Environmental awareness influences consumers' purchasing decisions and in turn the effects of environmental regulation. It is important then to take into account the differing perceptions of environmental quality of goods in order to design appropriate regulatory measures.

This paper investigates how consumers' perception influences the firms' investment

[1] For instance most of the products that make the claim are biodegradable, phosphate and chlorine free may still derive their ingredients from petrochemicals like crude oil or natural gas (which are not renewable). https://www.gmaonline.org/downloads/research-and-reports/greenshopper09.pdf

[2] The tools used in greenwashing can include press releases about green projects or task forces put into place, energy reduction or pollution reduction efforts, and rebranding of consumer products and advertising materials.

[3] Lyon and Maxwell (2011) analyse greenwashing as a game between a firm and an activist. In their analysis, greenwashing implies that the firm selectively discloses the positive information about their environmental or social performance without full revealing negative information.

in environmental quality and environmental regulation. We present a model of regulation in the presence of asymmetric information on the environmental quality. Our framework is characterised by the heterogeneity of consumers due to environmental awareness. We model the difference in consumers' environmental concern as a difference in their level of information, which is taken as given. This implies that firms' effort to affect consumers' perception is taken as given too. This assumption aims to keep the focus on the paper on how consumers' perception influences the firms' decisions and regulation rather than on the determination of consumers' perception. Moreover, we assume that the overestimation of environmental quality is positively related to greenwashing since the investment in green advertising induces consumers to overstate the environmental quality of the product.

In the baseline model, we evaluate the unregulated equilibrium and examine the variation in terms of under- (over-) estimation of the true quality. The equilibrium is symmetric. Profits in equilibrium are increasing in the perception, while, the investment in green quality decreases the more the goods are substitutes. Indeed, a higher degree of substitutability implies higher competition. Thus firms react by softening their investment in environmental quality.

The paper considers next two government interventions. First we investigate the effects of a tax on polluting emissions. This policy increases the investment in environmental quality due to changes on firms' incentives. The idea is that a company invests in green quality not only to acquire green consumers, like in the baseline model, but also to reduce tax burdening. We investigate next the introduction of an endogenous tax. The level of optimal taxation increases with the marginal damage on emissions if the average perception of environmental quality is sufficiently high. Indeed, a higher perception of environmental quality implies larger demand. If the marginal damage of emission is high, then the increase in demand needs to be compensated by higher taxation.

The second intervention regards the environmental standard. Despite of the downward sloping firms' reaction functions in quality which imply negative strategic effects, the introduction of an environmental standard increases the perceived environmental quality even for consumers who underestimate quality. This effect rises their willingness to pay for the goods, while increasing firms' profits. Instead, an endogenous environmental standard bites if the overestimation of environmental quality of some consumers is high, leading firms to invest more in quality. Finally, if the marginal damage is relatively low, then the environmental standard increases with the level of quality overestimation. Hence the optimal standard represents an effective regulatory instrument against greenwashing.

We finally compare the two interventions in a numerical simulation. The environmental standard increases the actual level of quality more than the tax for lower levels of marginal damage. Conversely, a lower quality of the green products is ensured under the standard whenever the marginal damage of pollution is high.

The economic literature has recently analysed the presence of green consumers. A first group of papers focused on the impact of a higher consumers' consciousness on the market equilibrium and the associated social welfare (Eriksson, 2004 and Conrad, 2005). A second group dealt with the presence of green consumers interacts with the optimal environmental policy (Arora and Gangopadhyay, 1995, Cremer and Thisse, 1999, Moraga-Gonzales and Padron-Fumero, 2002, Lombardini-Riipen, 2005). Finally, the presence of green consumers has been examined in relation with socially responsible firms (Rodriguez-Ibeas, 2007, Garcia-Gallego and Georgantzis, 2009, Doni and Ricchiuti, 2013), or in determining the validity of the Porter hypothesis (Andrè *et al.*, 2009, Lambertini and Tampieri, 2012). The most common framework of these contributions is the pure vertically differentiated duopoly. Unlike these approaches, our framework considers both horizontal and vertical differentiation and goods differ in the degree of substitutability and in (environmental) quality. Moreover, the present paper contributes to the literature by introducing the misperception of green consumers and its effects in the private and regulated equilibrium.

A related paper is Liu *et al.* (2012), who examine the impact of consumers' environmental awareness on competition among the supply chain players. They exploit a two-stage Stackelberg game in three supply chain network structures. They find that, as consumers' environmental awareness increases, retailers and manufacturers with superior eco-friendly operations will benefit; interestingly, higher levels of retail competition may make manufacturers with low eco-friendly operations benefit from the increase of consumers' environmental awareness.

The remainder of the paper is organised as follows. Section 2 introduces the model. Section 3 shows the baseline results. Section 4 considers the regulated equilibrium through the two interventions. These are developed and compared in Section 4.1, 4.2 and 4.3, respectively. Concluding remarks are proposed in Section 5. All formal proofs can be found in the appendix.

## 2 The model

Consider an economy with two firms, 1 and 2, each producing one differentiated good. There is a number of consumers with mass normalised to 1. Their representative utility function, $U(x_1, x_2)$, is:[4]

$$U(x_1, x_2) = (\alpha + e_1)\, x_1 + (\alpha + e_2)\, x_2 - \frac{x_1^2 + 2\beta x_1 x_2 + x_2^2}{2} + c_0, \tag{1}$$

where $(\alpha + e_i)$ represents the environmental quality of good $i$, $\alpha$ is an exogenous parameter (a "minimum" level of quality) while $e_i$ is determined by firm $i$, $i \in \{1, 2\}$; $x_i$ denotes the representative consumer's quantity of good $i$; $c_0$ is the quantity of the numeraire good; finally, $\beta \in [0, 1]$ denotes the degree of substitutability between the two goods, with $\beta = 0$ identifying independent goods and $\beta = 1$ in case of perfect substitutes.[5] Under symmetric information, consumers are perfectly aware of the intrinsic environmental quality of goods 1 and 2.[6] For each $i \in \{1, 2\}$, utility maximisation with respect to $x_1$ and $x_2$ gives a certain demand function:

$$x_i(p_i, p_j, e_i, e_j) = \frac{\alpha(1-\beta) + (e_i - \beta e_j) - p_i + \beta p_j}{1 - \beta^2}, \; j \in \{1, 2\}, j \neq i. \tag{2}$$

The rise in consumers' ecological consciousness is constrained by the fact that they may not have sufficient information about the environmental quality of a product. We model this uncertainty as a case of imperfect information in detecting the environmental quality.

Environmental perceptions are exogenous and independent draws. The reasons for which consumers are more or less aware of environmental quality are thus left aside from the analysis. Of course this implies that firms' effort to affect consumers' perception is taken as given too. Firms can indeed influence environmental perception through advertising and eco-label, for instance. However, the focus of this paper is on the impact of consumers' perception in the firms' decision on the investment in environmental quality rather than on the determination of consumers' perception.

Taking consumers' perception as given has several implications. First, there is no communication between consumers differing in perception of environmental quality.

[4] See Häckner (2000).

[5] A similar procedure is proposed by Garella and Petrakis (2008). They analyse the effect of introducing a minimum quality standard when consumers are not perfectly informed of goods' quality.

[6] Note that quality enters in the intercept of the demand function. Symeonidis (2003) considers the alternative approach in which qualities alter the slope of the demand functions.

For instance, the current groups could be the result of previous communications among consumers. Second, there is no credible third party certification available or credible for every consumers. This implicit assumption may be interpreted with some consumers being skeptical towards some type of certification. Third, informed consumers do not incur any cost to be informed. The reason could be due to idiosyncratic preferences towards environmental awareness. Some consumers intrinsically care for environment and enjoy to keep themselves informed without any cost.

For $i \in \{1, 2\}$, only a proportion $\lambda \in (0, 1)$ of the population recognises the true quality $e_i$. In turn, a proportion of $1 - \lambda$ consumers does not recognise the true environmental quality of good $i$. With equal probability they believe it is a low level $e_0 < e_i$ or a high level $e_m > e_i$. The symmetry in the probabilities of the events is for the sake of simplicity. It has the advantage of keeping the analysis tractable to examine the impact of wrong perception of goods. The interpretation of a consumer with uncertain perception is therefore:

- $e_0 < e_i$: the consumer underestimates the environmental quality of good $i$;
- $e_m > e_i$: the consumer overestimates the environmental quality of good $i$.

Let us define $e_0$ and $e_m$ as the *underestimation* and the *overestimation* of environmental quality, respectively. In particular, $e_m$ can be interpreted as a measure of *greenwashing*. It implicitly infers the firm's practise of making unwarranted or overblown claims of sustainability or environmental friendliness in an attempt to gain market share. The higher the signal $e_m$, the larger the consumers' overperception of the environmental quality of the good. Modeling greenwashing as a consequence of the exogenous overestimation of consumers follows the assumption of exogenous perception of environmental quality. Since we do not model the formation of perception of environmental quality, in turn we do not model the firms' decisions on their greenwashing behaviour. In turn, this implies that there is not direct cost, threat of inspection or penalty for firms if greenwashing emerges.

The expected proportion of consumers that receive the correct information about the environmental quality of both goods is $\lambda^2$. Then, a proportion of $(1 - \lambda)^2$ of consumers is expected to receive wrong information about both goods, whereas $2\lambda(1 - \lambda)$ consumers are expected to perceive wrong information about at most one of the goods. Hence, there are four equiprobable realisations of perceptions pairs $(e_0, e_0)$, $(e_m, e_m)$, $(e_{0,}e_m)$ and $(e_m, e_0)$, for individuals misperceiving the quality of both goods. Also, there are two equiprobable realisations for consumers with misperception of good 1, namely $(e_{0,}e_2)$ , and $(e_m, e_2)$. Similar results for good 2.

According to the perceived differences on quality, the demand for good 1 is given by:

$$\begin{aligned} q_1 = {} & \lambda^2 x_1 (e_1, e_2) + \\ & \frac{\lambda (1-\lambda)}{2} [x_1 (e_1, e_m) + x_1 (e_0, e_2) + x_1 (e_1, e_0) + x_1 (e_m, e_2)] + \\ & \frac{(1-\lambda)^2}{4} [x_1 (e_0, e_0) + x_1 (e_m, e_0) + x_1 (e_0, e_m) + x_1 (e_m, e_m)] . \end{aligned} \tag{3}$$

We thus represent nine types of consumers: one group of fully aware consumers, four groups of partially aware ones and the last four groups of wrongly aware consumers. Substituting the demands (2) of each consumer type into (3) yields:

$$q_i (p_i, p_j, e_i, e_j) = \frac{(1-\beta) \left(\alpha + \frac{1}{2} (e_m + e_0)\right) + \lambda (e_i - \beta e_j) - p_i + \beta p_j}{1-\beta^2}. \tag{4}$$

From (4), we can define the the average value of quality misperception as $\overline{e} = \frac{1}{2} (e_m + e_0)$. The demand function of firm $i$ always increases with $\overline{e}$. Intuitively, consumers' appetite for green products leads firms to put forward the better ecological quality of a given product.

Consumer surplus $CS$ is computed as the weighted average of the aggregate utility of all consumers' types. For instance, the utility of the consumers who are fully aware of goods 1 and 2 quality is given by:

$$U (x_1, x_2) = (\alpha + e_1^*) x_1 + (\alpha + e_2^*) x_2 - \frac{(x_1^2 + x_2^2 + 2\beta x_1 x_2)}{2} - p_1^* x_1 - p_2^* x_2. \tag{5}$$

Similar for the other groups. The explicit computation of consumer surplus can be found in the appendix.

The supply side is rather standard. Firms compete by choosing the environmental quality level $e$ and their prices.[7] We normalise marginal costs of production to zero, while fixed costs are increasing in quality:

$$C_1 = e_1^2,\ C_2 = e_2^2. \tag{6}$$

[7] Note that the demand function takes into account both horizontal and vertical differentiation, while, the information technology is the same between firms.

Thus the profit of firm $i$ is:

$$\pi_i = p_i q_i (p_i, p_j) - C_i. \tag{7}$$

In the spirit of Arora and Gangopadhyay (1995), the maximum level of emission allowed in the production process is denoted as $\overline{E}$. Firm $i$'s investment in environmental quality reduces total emissions of an amount $e_i$. Hence the net level of emission is $E = \overline{E} - (e_1 + e_2)$, where $E > 0$. This assumption rules out the unrealistic case in which investing in green quality more than offsets pollution. Indeed although new technologies and cleaner fuel can help cut down emissions of pollutants into the atmosphere, they are not replacing the non-green conventional production eliminating definitively the environmental damage. As standard in the literature, the environmental damage is assumed as a quadratic function of emissions, $D = dE^2$, where $d$ represents the marginal damage of emissions.[8]

Thus social welfare is given by:

$$SW = \sum_i^{1,2} \pi_i + CS - D.$$

The timing of the game is as follows. In the first stage, firms choose the level of environmental quality. In the second stage, firms compete in prices. The equilibrium concept is the subgame perfect equilibrium by backward induction.

# 3 Baseline results

In the market stage, each firm $i$ maximises profits with respect to $p_i$. The equilibrium price is

$$p_i^* (e_i, e_j) = \frac{\left(2 - \beta - \beta^2\right) \left[\alpha + \overline{e} (1 - \lambda)\right] + \lambda \left[e_i \left(2 - \beta^2\right) - \beta e_j\right]}{4 - \beta^2}, \tag{8}$$

where $p_1^* = p_2^*$ if and only if $e_1 = e_2$. The equilibrium price of firm $i$ is decreasing in the quality level chosen by its rival, $e_j$. Intuitively, a rise in consumers' consciousness increases their willingness to pay for environmental quality and reflects a higher consumer's marginal utility when they buy a green product. When a rival's quality increases, the premium commanded by one's own quality level at equilibrium is monotonically reduced. Indeed, the whole demand function to firm $i$ is shifted down. This negative effect is higher the larger the degree of the substitutability between the two goods, $\beta$.

[8] For simplicity, we abstract away from spillovers in the industry.

Equilibrium quantities are consequently given by:

$$q_i^* \left(p_i^* \left(e_i, e_j\right), p_j^* \left(e_i, e_j\right)\right) = \frac{p_i^*}{1-\beta^2}. \tag{9}$$

Thus they have the same properties of prices with respect to $e_j$ and $\overline{e}$.

In the first stage, each firm $i$ maximises its profit with respect to its environmental quality $e_i$:

$$\max_{e_i} \pi_i = \frac{\left[p_i^* \left(e_i, e_j\right)\right]^2}{1-\beta^2} - C_i \left(e_i, e_j\right).$$

The first order condition yields the best reply function for firm $i$:

$$e_i \left(e_j\right) = e\left(0\right) - \frac{\lambda^2 \beta \left(2-\beta^2\right)}{\left(4-\beta^2\right)^2 \left(1-\beta^2\right) - \lambda^2 \left(2-\beta^2\right)^2} e_j, \tag{10}$$

where

$$e\left(0\right) = \frac{\lambda \left(2-\beta^2\right)\left(1+\beta\right)\left(1-\beta\right)\left[\left(1-\lambda\right)\left(e_m + e_0\right)/2 + \alpha\right]}{\left(4-\beta^2\right)^2 \left(1-\beta^2\right) - \lambda^2 \left(2-\beta^2\right)^2},$$

and the denominator is always positive.[9] A preliminary result can be summarised in the next lemma:

**Lemma 1** *Green qualities are strategic substitutes.*

A firm's marginal revenue is sensitive to a change in its own environmental quality relative to a change in the rival's. Given the values of $\lambda$ and $\beta$, the magnitude of quality difference indicates the degree to which consumers' perceive the differentiation among the two products, or inversely, how close strategic substitutes the products are from the firms' perspectives. Thus an increase of the environmental quality of the competitor increases the marginal return from investing in quality. When the degree of substitutability increases, the two products become more homogeneous (lower differentiation or strategically perfect substitutes) and the firms' profits would drop to a lower level. Solving the system of (10), the equilibrium qualities are symmetric:

$$e_i^* = \frac{\lambda \left(2-\beta^2\right)\left[\alpha + \left(e_m + e_0\right)\left(1-\lambda\right)/2\right]}{\left(2-\beta\right)^2 \left(1+\beta\right)\left(2+\beta\right) - \lambda^2 \left(2-\beta^2\right)^2}. \tag{11}$$

[9] See the appendix.

Condition $e_i^* > e_0$ holds for

$$e_0 < \widetilde{e}_0 \equiv \frac{\lambda\left(2-\beta^2\right)\left[e_m\left(1-\lambda\right)+2\alpha\right]}{2\left(2-\beta\right)^2\left(1+\beta\right)\left(2+\beta\right)+\lambda\left(1+\lambda\right)\left(2-\beta^2\right)}. \tag{12}$$

Eq. (12) highlights the role played by the underestimation of consumers' perception and their belief that a higher environmental quality on the upper bound, $e_m$, signals an increase in the total environmental quality of the production process. The solution concept is therefore satisfied whenever the underestimation of perception is not too high, otherwise is not feasible. Moreover this symmetric outcome arises when the degree of substitution is not too high. Note that the equilibrium outcome of the pure vertically differentiated duopoly model cannot realise as a limiting case of our model as $\beta$ tends to 1. The demand systems are quite distinct to the case with only vertical differentiation. In our system, each consumer buys a variable quantity of both goods, whereas she buys a single unit of one good in the purely vertical differentiation models.[10] In equilibrium, firm $i$'s prices and profits are respectively,

$$p_i^* = \frac{\left(1-\beta^2\right)\left(4-\beta^2\right)}{\lambda\left(2-\beta^2\right)}e_i^*, \tag{13}$$

and

$$\pi_i^* = \frac{\left(1-\beta^2\right)\left(4-\beta^2\right)^2-\lambda\left(2-\beta^2\right)^2}{\lambda^2\left(2-\beta^2\right)^2}\left(e_i^*\right)^2. \tag{14}$$

We are now in a position to examine the characteristics of the equilibrium. Begin the comparative statics by evaluating how a variation in the degree of substitutability influences the equilibrium quality. We obtain that:

**Lemma 2** *The investment in green quality decreases the more the goods are substitutes.*

Lemma 2 suggests that the equilibrium quality is directly related to the degree of product substitutability only if the extra cost of producing green quality is sufficiently low. The convenience of the investment places countervailing incentives on a firm's optimal level of quality. Lower margins decrease the immediate costs of production, but also reduce the opportunity to face a higher demand due to consumers' environmental consciousness.

[10] See Gabszewicz and Thisse (1979, 1980) and Shaked and Sutton (1982), *inter alia*.

Consider further the analysis of a variation in the average perception of environmental quality. Differentiating the equilibrium qualities,

$$\frac{\partial e_i^*}{\partial \overline{e}} = \frac{2\lambda\left(2-\beta\right)\left(2-\beta^2\right)\left(1-\lambda\right)}{4\left(2-\beta\right)\left[^2\left(1+\beta\right)\left(2+\beta\right)-\lambda^2\right]} > 0. \tag{15}$$

The uncertainty about the value of the quality of a good has directly implications with personal misperception. In particular, given (15) together with (9), (13) and (14), it follows that,

**Proposition 1** *Qualities, prices, quantities and profits in equilibrium increase in the average misperception of environmental quality.*

The chain of evaluations leading to an increase in profits, prices and quantities from an increase in the misperception is fairly straightforward. Indeed Proposition 1 implies that, the higher the average quality perceived by the non-environmentally aware consumers, the larger the demand for firm's product and the higher the firms' incentives to invest in environmental quality due to a higher price in equilibrium.

# 4 Regulatory interventions

Emissions taxes and environmental standards are widely considered as the most common policy instruments for regulation of environmental externalities. Taxes generally raise government revenue, while usually an environmental standard specifies with a certain degree of precision the actions that a firm should undertake to achieve certain objectives.[11]

An emission tax works through the market by imposing a higher cost per unit of production. Pollution is therefore priced by the tax inducing entrepreneurs to release less of it. By leaving polluters free to choose their optimal emission levels, they can use their own strategy in order to minimize their costs. The optimal process for polluters minimizes their total private costs by reducing emissions until the tax rate equals their marginal abatement cost. This could be determined by any combination of pollution abatement or green innovation as in Chang (2011).

The environmental standard requires firm's emission not to fall short of a given level. With perfect information and a definite quality standard, a firm has in principle the same total abatement costs as in the tax system, but unlike the tax system it does

[11] See Holland (2012).

not have to pay for abatement.[12] This result is confirmed in case of environmentally conscious consumers by Deltas *et al.* (2013).

From a public policy view, the net social benefits of the tax or a standard set at the socially efficient level of emissions are identical. What differs is the impact on the polluters. We observe that these results are not true in case of imperfect information on quality.[13] The aim of this section is therefore to point out whether these instruments and their impacts are indeed best in the presence of misperception of quality and green-washing (Lyon and Maxwell, 2011). For each intervention, we evaluate first the case in which regulation is exogenous, then we assume it being endogenously determined by the government with the aim of maximising social welfare. In particular, we suppose that the government pre-commits to its environmental policy. This implies that the optimal regulation does not react to the firms' decisions, but it is pre-determined.[14]

## 4.1 Tax on emissions

Begin by introducing a tax on emissions in the economy. Firm $i$'s profit function is given by:

$$\pi_i = p_i q_i \left(p_i, p_j\right) - C_i - tE, \tag{16}$$

where taxation is a linear function of emissions $E$, and $t > 0$ is the unit tax. In turn, social welfare becomes

$$SW = \sum\nolimits_i^{1,2} \pi_i + CS - D + T,$$

where $T = 2tE$ denotes total tax revenue. The market stage remains unchanged compared to the unregulated case. In the first stage, equilibrium qualities are:

$$e_i^{t*} = -\frac{t\left(2-\beta\right)^2\left(1+\beta\right)\left(2+\beta\right) - 2\lambda\left[\overline{e}^2\left(1-\lambda\right)+\alpha\right]\left(2-\beta^2\right)}{2\left(2-\beta^2\right)\left[\left(1+\beta\right)\left(2+\beta\right)-\lambda^2\right]}. \tag{17}$$

Eq. (17) underlines the role placed by the average misperception on quality level. In particular, it influences negatively the quality derived on tax emission. This result is still valid if the degree of substitution is high. Comparing the level of qualities with the

[12] See Bottega and De Freitas (2009).

[13] See Sengupta (2012) for an alternative model in the presence of misperception of quality of green consumers.

[14] We thus abstract away from the debate on time-inconsistency of environmental policies. See Petrakis and Xepapadeas (2001 and 2003), and Golombek *et al.* (2010), *inter alia*.

(previous) unregulated case we get,

$$e_i^* - e_i^{t*} = -\frac{t\,(2-\beta)^2\,(1+\beta)\,(2+\beta)}{2\left(2-\beta^2\right)\left[(1+\beta)\,(2+\beta)-\lambda^2\right]} < 0. \quad (18)$$

Therefore,

**Proposition 2** *The introduction of a tax on polluting emission increases the investment in environmental quality.*

By introducing a Pigouvian tax, the objective of investing in environmental quality becomes two-fold: a firm invests in green quality in order to acquire green consumers, meanwhile, the investment reduces fiscal costs.

Now suppose that there is a pre-stage where government sets a Pigouvian tax with the aim to maximise social welfare. The first order condition of $SW$ with respect to $t$ yields the socially optimal tax $t^*$(see the appendix). The result can be summarised as follows:

**Proposition 3** *The level of optimal taxation (see the appendix for definitions of* $\widetilde{e}$ *and* $\widetilde{d}$*):*

- *increases with the marginal damage of emissions when* $\overline{e} > \widetilde{e}$.
- *decreases with the average perception of environmental quality when* $d > \widetilde{d}$ .

The results of Proposition 3 can be explained as follows. At the optimum, with high marginal damage of emission, the increase of demand needs to be compensated by higher taxation. Nonetheless, a high perception of environmental quality induces greater demand functions such that a lower level of taxation is sufficient to reach the optimum.

The last result intuitively suggests how the optimal tax reacts to the presence of greenwashing by the analysis of a variation in $e_m$. Differentiating $t^*$ with respect to $e_m$ (see the appendix) yields:

$$\frac{\partial t^*}{\partial e_m} < 0, \quad (19)$$

for $d > \widetilde{d}$. The intuition is similar too: a higher level of greenwashing stimulates demand, hence a lower tax is enough to reach the optimum.

### 4.2 Environmental standard

In this section we introduce an environmental standard that sets the minimum level of environmental quality. It is generally defined by regulation which specifies the maximum permissible concentration of a potentially hazardous chemical in an environmental sample. We denote it as $\widehat{e} > e_0$, i.e., a predetermined value higher than the lower bound in misperception.[15] Whenever $\widehat{e} > e_0$, eco-friendly consumers, who receive the low quality signal for good $i$, revise their beliefs and update it to $e_0 = \widehat{e}$. This increases their willingness to pay for that product. Begin by evaluating how this influences the quality investment in equilibrium. Differentiating $e_1^*$ and $e_2^*$ with respect to $e_0$, it yields:

$$\frac{\partial e_i^*}{\partial e_0} = \frac{\lambda\left(2-\beta^2\right)(1-\lambda)}{2\left(2-\beta\right)^2\left[(1+\beta)(2+\beta)-2\lambda^2\right]} > 0. \tag{20}$$

This result implies that the environmental standard has a positive impact on firms' qualities satisfying the higher need of green products.

**Proposition 4** *Introducing an environmental standard increases the quality investment of both firms.*

The higher willingness to pay of consumers in turn affects the firms' incentives to invest in environmental quality. The implementation of a standard guarantees a higher level of green type due to the exogenous threshold. With regards to the effect of substitutability among goods, we differentiate with respect to $\beta$ such that:

$$\frac{\partial e_i^*}{\partial e_0 \partial \beta} = -\frac{\lambda\left(8-8\beta-2\beta^2+8\beta^3+\beta^4-2\beta^5\right)(1-\lambda)}{2\left[8+4\beta-\beta^3+\beta^4-2\lambda^2-\beta^2\left(6-\lambda^2\right)\right]^2} < 0. \tag{21}$$

This implies that the substitutability among goods, $\beta$, reduces the impact of this policy in the market. The lack of strategic effects imply that, even if the environmental standard succeeds in inducing cleaner products, its efficacy depends on products substitutability. Moreover, the higher willingness to pay for green products induces firms to invest in higher quality and green innovation. This effect is still higher, the more the goods are substitutes. Let us evaluate how the impact of the environmental standard influences qualities, prices, quantities and profits. According to Proposition 1 and observing the role of the standard, it follows that:

[15] This procedure is borrowed from Garella and Petrakis (2008).

**Proposition 5** *Qualities, prices, quantities and profits rise in equilibrium if an environmental standard, $\widehat{e}$, is introduced such that $\widehat{e} > e_0$.*

Since the consumers' willingness to pay is increased with respect to the unregulated case, their demand for both goods shifts up. Firms offer goods of higher quality, so that they can also charge higher prices. In turn profits increase.

Consider next the introduction of an endogenous environmental standard.[16] Suppose that there is a pre-stage in which the government sets an environmental standard $\widehat{e} > e_0$ so as to maximise social welfare. Consumers will update their evaluation of the lower bound of environmental quality, so that $e_0^* = \widehat{e}$. The first order condition with respect to $e_0$ yields (see the appendix) $e_0^*$.

The first question is whether or not the introduction of an optimal environmental standard would bite, i.e., if it indeed affects the level of investment in environmental quality of the firm or not. Comparing $\widehat{e}^*$ with $e_i^*$, i.e., the equilibrium quality in the unregulated case yields:

$$e_0^* - e_i^* > 0,$$

for

$$e_m > \widetilde{e}_m, \tag{22}$$

where $\widetilde{e}_m$ is defined in the appendix. Hence

**Proposition 6** *An optimal environmental standard bites only if the overestimation of the environmental quality $e_m$ is sufficiently high.*

Proposition 6 shows how the perception of environmental quality may affect the effectiveness of a policy based on environmental standards. In particular, when consumers' overestimation is high, then demands increase even in case of low perception. Indeed, firms are willing to invest less in environmental quality since consumers overestimate it considerably. In this case, the introduction of an optimal environmental standard induce firms to invest more in quality, and therefore, it is an effective regulatory tool in industries where greenwashing is adopted. Finally, note that there is no trade-off between consumer surplus and damage function. This is due to the fact that the quality is green and consumers internalise it in their utility function, so that incentives are aligned. This effect does not emerge in situations where quality is hedonic rather than green. In this case, an increase in consumer surplus would imply higher emissions (Lambertini and Tampieri, 2012 and Ecchia *et al.*, 2013).

[16] See Ecchia and Lambertini (1997) for an analysis of endogenous minimum quality standard.

We then examine the introduction of an optimal environmental standard on greenwashing. Differentiating $e_0^*$ with respect to $e_m$ yields:

$$\frac{\partial e_0^*}{\partial e_m} > 0, \tag{23}$$

for $d < \widehat{d}$ (see the appendix for eq. (23) and the definition of $\widehat{d}$). Equation (23) suggests that, if the marginal damage is relatively low, then the overestimation of the environmental quality increases the level of the optimal environmental standard. This also guarantees an effective policy by the government, since the optimal standard reacts to the level of greenwashing. The effect emerges only if the emissions are not so harmful or relatively limited over time. Otherwise, the presence of greenwashing reduces the endogenous threshold motivated by lower expected returns.

## 4.3 Tax vs standard

We now propose a simple simulation to examine the impact of the two optimal policies on quality levels. Unlike the case of perfect information, where the effect on qualities is analogous between the two interventions (Holland, 2012), we show that the misperception in qualities determines a different effect of the emission tax and the standard.

We compare the equilibrium qualities when either the optimal tax or the optimal standard are applied. We allow the quality levels to change according to the marginal damage of emission $d$. We assume generic values of the parameters of the model: the proportion of consumers that recognise the quality of one good ($\lambda$) is equal to 0.1; the substitutability among goods, $\beta$, is assumed to be 0.3; while, the constant coefficient $\alpha$ is 1. Finally, since introducing the emission standard does not affect qualities whenever the overestimation is relatively low (see Proposition 6), we set the conditions on $e_m$ such that the environmental standard is biting.

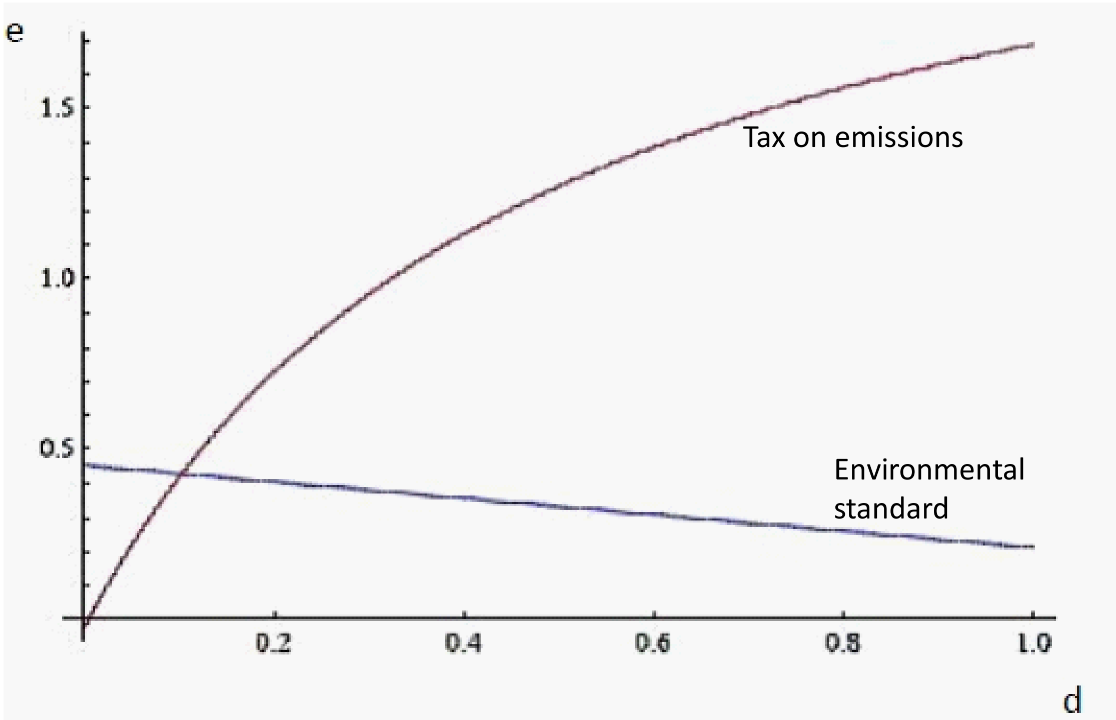


Figure 1 summarises the results. Equilibrium quality in the presence of optimal taxation is an increasing and concave function of marginal damage of emission, while the equilibrium quality is decreasing and concave with environmental standard. Both interventions exert an ongoing pressure on price competition and reduce the output of the not eco-friendly products. For lower level of damage, the environmental standard with misperception ensures higher level of quality. The results are not confirmed for higher value of $d$, where a tax on emission guarantees higher level of environmental quality. This shift occurs since an emission tax imposes a higher unit of production cost. Its impact expands for higher level of emissions, rising abatement costs.

# 5 Concluding remarks

In a world where green consumers misperceive the environmental quality of products and the consequent damage caused by firm's production, we have analysed how misperception influences the incentive to invest in cleaner production as regulatory interventions. The heterogeneity of information that agents can receive guarantees higher profits for firms, and the effect is higher the larger the impact of greenwashing in the market. The introduction of a tax on emissions rises the investment in environmental quality due to the higher cost of production, and quality always increases. An optimal environmental standard can be effective against greenwashing, conditional to a low marginal damage of emissions.

A possible extension of the present analysis may take into account asymmetric technologies among firms. The idea is that the importance of environmental perception is reduced whenever one firm is more efficient than the competitor. This proxies the market power of the efficient firm with higher profits as quality increases. Accordingly, the weight of overestimation (and in turn greenwashing) in determining quality is relatively lower than in the symmetric case, since profits for the efficient firm grow more than its less efficient competitor. By the same token, the level of the average value of quality misperception has a lower weight for the efficient firm than for the inefficient one. It is higher in the presence of an environmental standard rather than with a tax on emission, since the former policy does not affect production costs. These impacts could be so large under the standard relative to the tax that outweighs the reduction in the environmental quality of the products. The analysis with asymmetric technology among firms is left for future research.

# References


[1] André, F.J., González, P. and Porteiro, N. 2009. Strategic quality competition and the Porter hypothesis. *Journal of Environmental Economics and Management* **57**: 182-194.

[2] Arora, S. and Gangopadhyay, S. 1995. Toward a theoretical model of voluntary overcompliance. *Journal of Economic Behavior and Organization* **28**: 289-309.

[3] Bansal, S and Gangopadhyay, S. 2003. Tax/subsidy policies in the presence of environmentally aware consumers. *Journal of Environmental Economics and Management* **45**: 333-355.

[4] Bottega, L. and De Freitas, J. 2009. Public, private and nonprofit regulation for environmental quality. *Journal of Economics and Management Strategy* **18**: 105-123.

[5] Chang, C.H. Consumers keen on green but marketers don't deliver. *Advertising Age* **63**: 2-4.

[6] Chang, C.H. 2011. The influence of corporate environmental ethics on competitive advantage: The mediation role of green innovation. *Journal of Business Ethics* **104**: 361-370.

[7] Chase, D. and Smith, T.K., 1992. Competitive investment in clean technology and uninformed green consumers. *Auburn Economics Working Paper Series auwp2012-08.*

[8] Chen, Y.S. 2008. The driver of green innovation and green image - Green core competence. *Journal of Business Ethics* **81**: 531-543.

[9] Chen, Y.S. and Chang, C.H. 2013. Greenwash and green trust: the mediation effects of green consumer confusion and green perceived risk. *Journal of Business Ethics* **114**: 489-500.

[10] Conrad, K. 2005. Price competition and product differentiation when consumers care for the environment. *Environmental and Resource Economics* **31**: 1-19.

[11] Cremer, H. and Thisse, J.F. 1999. On the taxation of polluting products in a differentiated industry. *European Economic Review* **43:** 575-594.

[12] Deltas G., Harrington D. R., Khanna M., 2013. Oligopolies with (somewhat) environmentally conscious consumers: Market Equilibrium and Regulatory Intervention. *Journal of Economics and Management Strategy* **22**: 640-667.

[13] Doni, N. and Ricchiuti, G. 2013. Market equilibrium in the presence of green consumers and responsible firms: A comparative statics analysis. *Resource and Energy Economics* **35**: 380-395.

[14] Ecchia, G., Lambertini, L. 1997. Minimum quality standards and collusion, *Journal of Industrial Economics* **45:** 101-113

[15] Ecchia, G., Lambertini, L. and Tampieri, A. 2013. Minimum quality standards in hedonic markets with environmental externalities. *Environmental Modeling and Assessment* 18: 319-323.

[16] Eriksson, C. 2004. Can green consumerism replace environmental regulation? A differentiated-products Examples. *Resource and Energy Economics* **26:** 281-293.

[17] Garella, P. G., and Petrakis, E. 2008. Minimum quality standards and consumers' information. *Economic Theory* **36**: 283-302.

[18] Garcia-Gallego, A. and Georgantzís, N. 2009. Market effects of changes in consumers' social responsibility. *Journal of Economics and Management Strategy,* **19:** 453-487.

[19] Golombek, R., Greaker, M., and Hoel, M. 2010. Climate Policy without Commitment. *CESifo WP 2909.*

[20] Häckner, J. 2000. A note on price and quantity competition in differentiated oligopolies. *Journal of Economic Theory* **93**: 233-239.

[21] Holland, S. 2012. Emissions taxes versus intensity standards: Second-best environmental policies with incomplete regulation. *Journal of Environmental Economics and Management* **63**: 375-387.

[22] Kim, Y. and Choi, S.R. 2005. Antecedents of green purchase behaviour: An examination of collectivism, environmental concern and PCE. *Advances in Consumer Research* **32**: 592-599.

[23] Kohl, M. F., 1991. *Good Earth Art.* Bellingham: Bright Ring Publishing.

[24] Lambertini, L. and Tampieri, A. 2012a. Do minimum quality standards bite in polluting industries? *Research in Economics* **66:** 184-194.

[25] Lambertini, L. and Tampieri, A. 2012b. Vertical differentiation in a Cournot industry: The Porter hypothesis and beyond. *Resource and Energy Economics* **34:** 374-380.

[26] Liu, Z., Anderson, T.D. and Cruz, J.M. 2012. Consumer environmental awareness and competition in a two-stage supply chains. *European Journal of Operational Research* **218**: 602-613.

[27] Lyon, T.P. and Maxwell J. W. 2011. Greenwash: Corporate environmental disclosure under threat of audit, *Journal of Economics and Management Strategy* **20**: 3-41

[28] Lombardini-Riipinen, C. 2005. Optimal tax policy under environmental quality competition. *Environmental and Resource Economics* **32**: 317-336.

[29] Moraga-Gonzalez, J. L. and Padron-Fumero, N. 2002. Environmental policy in a green market. *Environmental and Resource Economics* **22**: 419-447.

[30] Petrakis, E. and Xepapadeas, A. 2001. To commit or not to commit: Environmental policy in imperfectly competitive markets. *University of Crete Working Paper* **110.**

[31] Petrakis, E. and Xepapadeas, A. 2003. Location decisions of a polluting firm and the time consistency of environmental policy. *Resource and Energy Economics* **25**: 197-214.

[32] Reitman, V. 1992. Green products sales seem to be wilting. *The Wall Street Journal* **220:** 18.

[33] Rodriguez-Ibeas, R. 2007. Environmental product differentiation and environmental awareness. *Environmental and Resource Economics* **36**: 237-254.

[34] Sengupta, A. 2012. Investment in cleaner technology and signaling distortions in a market with green consumers, *Journal of Environmental Economics and Management* **64:** 468-480

# 6 Appendix

## 6.1 Section 2

### 6.1.1 Consumer surplus

By evaluating the sum of utilities, consumer surplus is:

$$\begin{aligned} CS &= \frac{1}{1-\beta^2}\left[2\left(p_1^{*2}+p_2^{*2}+e_m^2\right)+4\alpha\left(1-\beta\right)\left(\alpha+e_m+e_0\right)-2p_2^*\left(2\alpha+e_m+e_0\right)\left(1-\beta\right)+\right. \\ & -\beta\left(e_0+e_m\right)^2-2p_2^*\lambda\left(2e_2^*-\left(e_m+e_0\right)\left(1-\beta\right)-2e_1\beta\right)+ \\ & -2\lambda\left(\left(e_m+e_0-e_1^*-e_2^*\right)\left(e_m+e_0-e_1^*-e_2^*-\beta\left(2\alpha+e_m+e_0\right)-2\alpha\right)\right)+ \\ & -\lambda^2\beta\left(e_m+e_0-2e_1^*\right)\left(e_m+e_0-2e_2^*\right) \\ & \left.-2p_1^*\left(\left(e_m+e_0\right)\left(1-\beta\right)\left(1-\lambda\right)+2\left(\alpha+p_2^*\beta+e_1^*\lambda-\alpha\beta-e_2^*\lambda\beta\right)\right)\right]. \end{aligned}$$

## 6.2 Section 3

### 6.2.1 Proof of Lemma 1

The second order conditions of $\pi_i$ with respect to $e_i$ yields:

$$\frac{\partial^2\pi_i}{\partial e_i^2}=\frac{2\lambda^2\left(2-\beta^2\right)^2}{\left(4-\beta^2\right)^2\left(1-\beta^2\right)}-2<0,$$

for

$$\lambda^2<\widehat{\lambda}^2\equiv\frac{\left(4-\beta^2\right)^2\left(1-\beta^2\right)}{\left(2-\beta^2\right)^2}. \tag{24}$$

Thus the problem admits a maximum when the group of fully aware consumers is not too large. Note also that $\widehat{\lambda}^2(1)=0$, implying that the equilibrium requires some substitutability among goods. Condition (24) implies the positivity of best reply (10) and is sufficient condition in order to get substitutability in quality investments. ■

### 6.2.2 Proof of Lemma 2

Differentiating $e_i^*$ with respect to $\beta$ yields:

$$\frac{\partial e_i^*}{\partial \beta} = \frac{\lambda\left(8-8\beta-2\beta^2+8\beta^3+\beta^4-2\beta^5\right)\left[\left(e_m+e_0\right)(1-\lambda)/2+2\alpha\right]}{2\left[8+4\beta-\beta^3+\beta^4-2\lambda^2-\beta^2\left(6-2\lambda^2\right)\right]^2} < 0. \quad \blacksquare$$

## 6.3 Section 4.1

### 6.3.1 Characterization of the Endogenous Tax

The first order condition of the social welfare function $SW$ with respect to $t$ yields the optimal level of taxation:

$$t^* = A + \overline{e}B,$$

where

$$A = \frac{4d\overline{e}\left(2+\beta-\beta^2\right)^2\left(2-\beta-\beta^2\right)+2\alpha\lambda\left(1-\beta^2\right)\left[2-\beta-4d\left(2-\beta^2\right)\right]+2\lambda^2\left(2-\beta^2\right)\left[\alpha-2d\overline{e}\left(1-\beta^2\right)\right]+2\alpha\lambda^3}{2\left(1+2d\right)\left(2+\beta-\beta^2\right)^2\left(2-\beta-\beta^2\right)-\lambda\left(2-\beta\right)^2\left(1+\beta\right)\left(2+\beta\right)+\lambda^2\left(1+\beta\right)\left(2+\beta\right)\left[1+\beta\left(1-\beta\right)\right]}$$

$$B = \frac{2\lambda\left(1-\lambda\right)\left[6+4d\left(2-3\beta^2+\beta^4\right)-2\lambda+\beta\left(5-\beta\left(4+\beta\left(2-\beta\right)-\lambda\right)\right)\right]}{2\left(1+2d\right)\left(2+\beta-\beta^2\right)^2\left(2-\beta-\beta^2\right)-\lambda\left(2-\beta\right)^2\left(1+\beta\right)\left(2+\beta\right)+\lambda^2\left(1+\beta\right)\left(2+\beta\right)\left[1+\beta\left(1-\beta\right)\right]} \quad \blacksquare$$

### 6.3.2 Proof of Proposition 3

Begin by examining the variation of the optimal tax to a change in the marginal damage of emissions. Differentiating $t^*$ with respect to $d$ yields:

$$\frac{\partial t^*}{\partial d} \propto 8\left(1-\beta\right)\left(1+\beta-\beta^2\right)\left(1-\lambda\right)\left[8+4\beta-\beta^3+\beta^4-2\lambda^2-\beta^2\left(6-\lambda^2\right)\right]\overline{e}+$$
$$-4\lambda\left(1-\beta\right)\left[8+4\beta-\beta^3+\beta^4-2\lambda^2-\beta^2\left(6-\lambda^2\right)\right]\left[2\alpha\lambda\left(3-5\beta+2\beta^2\right)\right]+$$
$$-\overline{e}\left[8+8\beta^3-2\beta^4-4\lambda+\lambda^2+\beta^2\left(6+\lambda+\lambda^2\right)+\beta\left(8-4\lambda-\lambda^2\right)\right] > 0,$$

if and only if

$$\overline{e} > \widetilde{e}$$

where

$$\widetilde{e} \equiv \frac{4\lambda\left(1-\beta\right)\left[8+4\beta-\beta^{3}+\beta^{4}-2\lambda^{2}-\beta^{2}\left(6-\lambda^{2}\right)\right]\left[2\alpha\lambda\left(3-5\beta+2\beta^{2}\right)\right]}{8\left(1-\beta\right)\left(1+\beta-\beta^{2}\right)\left(1-\lambda\right)\left[8+4\beta-\beta^{3}+\beta^{4}-2\lambda^{2}-\beta^{2}\left(6-\lambda^{2}\right)\right]}$$

$$+\frac{\overline{e}\left[8+8\beta^{3}-2\beta^{4}-4\lambda+\lambda^{2}+\beta^{2}\left(6+\lambda+\lambda^{2}\right)+\beta\left(8-4\lambda-\lambda^{2}\right)\right]}{8\left(1-\beta\right)\left(1+\beta-\beta^{2}\right)\left(1-\lambda\right)\left[8+4\beta-\beta^{3}+\beta^{4}-2\lambda^{2}-\beta^{2}\left(6-\lambda^{2}\right)\right]}$$

We investigate next how a change in the perception of environmental quality would influence the optimal level of taxation. By differentiating $t^*$ with respect to the average perception of environmental quality, it yields:

$$\frac{\partial t^*}{\partial \overline{e}} = \frac{2\lambda\left(1-\lambda\right)\left[6+4d\left(2-3\beta^{2}+\beta^{4}\right)+2\lambda-\beta\left(5-\beta\left(4+\beta\left(2-\beta\right)-\lambda\right)\right)\right]}{\left[\lambda\left(2-\beta\right)^{2}-\lambda^{2}\left[1-\beta\left(1-\beta\right)\right]\right]\left(1+\beta\right)\left(2+\beta\right)-2\left(1+2d\right)\left(2+\beta-\beta^{2}\right)^{2}\left(2-\beta-\beta^{2}\right)} < 0$$

if and only if,

$$d > \widetilde{d} \equiv \frac{\lambda\left[4-\lambda-\beta\left(4+\lambda-\beta\left(1+\lambda\right)\right)\right]}{4\left(2-\beta\right)^{2}\left(1-\beta^{2}\right)} - \frac{1}{2}$$

The cross derivative of $t^*$ with respect to $\overline{e}$ and $d$ confirms the results above:

$$\frac{\partial t^*}{\partial \overline{e}\partial d} \propto \left(2-\beta\right)^{2}\left(1+\beta\right)\left(2+\beta\right)-\lambda^{2}\left(2-\beta^{2}\right) > 0 \qquad \blacksquare$$

### 6.3.3 Proof of eq. (19)

Differentiating $t^*$ with respect to $e_m$ yields:

$$\frac{\partial t^*}{\partial e_m} = -\frac{\lambda\left(1-\lambda\right)\left[6+4d\left(2-3\beta^{2}+\beta^{4}\right)-2\lambda+\beta\left(5-\beta\left(4+2\beta\left(2-\beta\right)+\lambda\right)\right)\right]}{\left(1+\beta\right)\left(2+\beta\right)\left[\lambda\left(2-\beta\right)^{2}-\lambda^{2}\left(1+\beta\left(1-\beta\right)\right)\right]-2\left(1+2d\right)\left(2+\beta-\beta^{2}\right)^{2}\left(2-\beta-\beta^{2}\right)} > 0,$$

for $d < \widetilde{d}$.

## 6.4 Section 4.2

### 6.4.1 Characterization of the Endogenous Standard

The first order condition of $SW$ with respect to $t$ yields the optimal level of taxation:

$$e_0^* = C + e_m F, \tag{25}$$

such that

$$C = \frac{2}{\left(2-\beta^2\right)} \frac{\Phi - \Psi}{\Lambda - \Gamma} \quad \text{and} \quad F = \frac{1}{\left(2-\beta^2\right)} \frac{\digamma}{\Lambda - \Gamma}$$

where variables $\Phi$, $\Psi$, $\Lambda$, $\Gamma$, $\digamma$ respectively indicating,

$$\begin{aligned}
\Phi &= 2d\lambda\bar{e}\left(1-\beta\right)\left(2-\beta^2\right)\left(2+\beta\right)\left(2+\beta-\beta^2\right)^2\left(2-\beta^2\right) - \alpha\left(2-\beta\right)^2\left(1-\beta\right)\left(1+\beta\right)^2\left(2+\beta\right)^2\left(3-2\beta\right) \\
\Psi &= \alpha\lambda^2\left(1+\beta\right)\left(2-\beta^2\right)\left[12+4d\left(1-\beta\right)\left(2-\beta^2\right)-\beta\left(8-\beta\left(4-3\beta\right)\right)\right] - \lambda^3\left(2-\beta^2\right)^2\left[\alpha-2d\bar{e}\left(1-\beta^3\right)\right] \\
\Lambda &= 2\lambda^2\left(1+\beta\right)\left[18+2d\left(1-\beta\right)\left(2-\beta^2\right)-\beta\left(10-\beta\left(5-3\beta\right)\right)\right] - \left(1+\beta\right)^2\left[7-3\beta\left(3-\beta\right)+\lambda\left(1+\beta\left(1-\beta\right)\right)\right] \\
\Gamma &= \lambda^3\left[3-2\beta^2+4d\left(1-\beta^2\right)\right]+\lambda^4 \\
\digamma &= 2\lambda^2\left(1+2d\right)\left(2-\beta\right)^2\left(1-\beta^2\right) - \left(1-\lambda\right)\left[\left(2-\beta\right)^2\left(1+\beta\right)^2\left(2+\beta\right)^2\left(1+\beta\left(1-\beta\right)\right)\right] - \lambda^3\left(2-\beta^2\right)^2 \quad \blacksquare
\end{aligned}$$

### 6.4.2 Proof of eq. (23):

Given eq. (25), it can be easily shown that $\frac{\partial e_0^*}{\partial e_m} = F$. This is positive if and only if:

$$d < \widehat{d} \equiv \frac{\left(2-\beta\right)^2\left(1+\beta\right)^2\left(2+\beta\right)^2\left[7-3\beta\left(3-\beta\right)+\lambda\left(1+\beta\left(1-\beta\right)\right)\right]}{4\lambda^2\left(2-\beta^2\right)^2\left(1-\beta^2\right)\left(1-\lambda\right)} + \tag{26}$$

$$+\frac{\lambda^3\left(3-2\beta^2\right)\left(2-\beta^2\right)^2 - 2\lambda^2\left(1+\beta\right)\left(2-\beta^2\right)\left[18-\beta\left(10+\beta\left(5-3\beta\right)\right)\right]}{4\lambda^2\left(2-\beta^2\right)^2\left(1-\beta^2\right)\left(1-\lambda\right)} \quad \blacksquare$$

### 6.4.3 Proof of Proposition 6

It is enough to prove that

$$e_m > \widetilde{e}_m \equiv \frac{\chi + \psi}{\varrho + \vartheta} \tag{27}$$

where now all the variables $\chi$,$\psi$, $\varrho$,$\vartheta$ respectively indicate

$$\begin{aligned}
\chi &= 2\alpha\left(1-\beta\right)\left(1+\beta\right)\left(3-2\beta\right)\left(4-\beta^2\right)^2 + \lambda\left(2-\beta\right)^2\left(2+\beta\right)\left(2-\beta^2\right)\left[\alpha+4d\bar{e}\left(1-\beta^2\right)\right] \\
\psi &= \lambda^2\left(2-\beta^2\right)\left[2d\left(\bar{e}+4\alpha\right)\left(1-\beta\right)\left(2-\beta^2\right)+\alpha\left[16-\beta\left(12-\beta\left(6-5\beta\right)\right)\right]\right] - 2d\bar{e}\lambda^3\left(1-\beta\right)\left(2-\beta^2\right)^2 \\
\varrho &= \left(1+2\beta-\beta^3\right)\left(4-\beta^2\right)^2 - \lambda^2\left(2-\beta\right)^2\left(2+\beta\right)\left[4+\beta\left(5-\beta\left(1-\beta\left(2+\beta\right)\right)\right)\right] \\
\vartheta &= \lambda^2\left(2-\beta^2\right)\left[12+4d\left(1-\beta\right)\left(2-\beta^2\right)-\beta\left[8-\beta\left(4-3\beta\right)\right]\right] - 2\lambda^3\left(1+2d\right)\left(1-\beta\right)\left(2-\beta^2\right)^2 \quad \blacksquare
\end{aligned}$$