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Transboundary Pollution Abatement: The Impact of Unilateral Commitment in Differential Games *

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Abstract. The present study explores the strategic interactions of countries setting pollution abatement policies in a dynamic two-player game. To reach a common target of environmental quality, countries can choose to commit to a stream of pollution abatement right from the beginning of the game or decide upon abatement at each moment in time. Most of the literature studies homogenous strategies, where no country or all countries commit to a (same) predefined policy. The main novelty of this paper resides in the introduction of heterogeneous strategies, where only one country commits to a specific abatement policy and which is actually the kind of strategic behavior currently observed among large pollution nations. We find that the pollution level can be lower under heterogenous than under homogenous strategies. A stringent environmental quality target will induce the committed player to produce an abatement effort that more than compensates the free-riding attitude of the non-committed player.

Keywords: Heterogeneous strategies, differential games, transboundary pollution, abatement.
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1 Introduction

The present paper studies whether unilateral action to cut greenhouse gas (GHG) emissions necessarily leads to a higher global pollution level than a coordinated behavior among countries. To address this question we develop a finite-horizon differential game and apply the concept of *heterogeneous strategies*, which allows one player to follow an open-loop strategy, while the other player follows a Markovian strategy. In addition to delivering a stationary solution, this concept allows us to study the transitional dynamics of the game.

Thus far, the potential consequences of unilateral action in climate regulation have been largely neglected in the dynamic games literature. The latter considers essentially cases in which both players follow *homogeneous strategies*, i. e., strategies where either both players play open-loop strategies or both play Markovian strategies.[1] In fact, ever since the critique of open-loop strategies by van der Ploeg and de Zeeuw (1992) and Dockner and Sorger (1996), the dominant approach has been one in which both players adopt a Markovian strategy. According to these authors, open-loop strategies lack realism and underestimate the negative effects of uncoordinated actions. They conclude that open-loop strategies in environmental games have only a role as a benchmark. More recently, Long (2010, p. 3) follows the same line reckoning that 'in analyzing dynamic games, one should try where possible to find an equilibrium in Markov-perfect (feedback) strategies'.

Since climate change is a global problem and since in order to be effectively addressed it requires joint action of many (if not all) countries, the discussion in public and academic circles has predominantly revolved around the question of an appropriate design of an international agreement. The choice to model dynamic environmental games of emission reduction with homogeneous strategies may therefore be attributed to an implicit focus on an international agreement as the only possible answer to the global climate problem. From this perspective a homogeneous strategy where both players play open-loop would coincide with a world in which a binding international agreement exists - a scenario that despite the numerous international climate conferences has not been attained yet.[2] By contrast, a homogeneous strategy where both players play Markovian strategies has been viewed to best reflect the current situation

[1] See Long (2010) for a complete survey about the application of differential games in economics studies. And the early contribution about choice of strategy space can be found in the seminal paper of Reinganum and Stokey (1985).

[2] Reasons for the lack of efficient action so far can be traced back to a number of difficulties in setting up efficient policies in light of climate change. Many of those that may have contributed to the dead end of negotiations so far have been highlighted by experts from the IPCC (IPCC (1995), Aflaki (2013)) , as e. g., disagreements on the long term consequences of GHG emissions on climate (and the ensuing costs) or the asymmetric divide of costs of climate change and benefits of mitigation thereof.

where no such international agreement exists.

However, in recent years several countries have announced commitments to unilateral action to cut emissions. The European Union (EU) heads of state and government, at the European Council meeting in March 2007, emphasized the EU's determination to unilaterally reduce its emissions. More precisely, the EU country representatives committed to a unilateral 20 per cent (resp. 40 per cent) reduction in GHG emissions compared to 1990 levels by 2020 (resp. 2030). The EU's unilateral commitment to reduce emissions has been adopted by its legislation. Similar measures have been announced by several other countries. For example, Australia's government commits to unconditionally reducing its emissions by 5 per cent compared to 2000 levels by 2020, and 15 per cent by 2020 if there is a global agreement under which major developing economies commit to substantially restraining their emissions and advanced economies take on commitments comparable to Australia's. More recently, China and the US (the two largest polluter in the world) have come to an agreement, where the US would cut its emissions 26 to 28 per cent below their 2005 levels by 2025, and China commits to get 20 per cent of its energy from non-fossil-fuel sources by 2030, and to peak its overall carbon dioxide emissions that same year.

In light of these recent developments, the aforementioned use of a homogeneous strategy appears ill-suited to study the consequences of unilateral commitment in this kind of collective action problem. The present paper contributes to the literature by explicitly addressing the fact that countries may take unilateral actions and analyzing the consequences thereof in a stylized model.

To that purpose, we set up a two-player finite-horizon differential game and allow for different strategic behavior in terms of pollution abatement policies. In particular, each player (country) chooses the pollution abatement, i. e., emission reduction, that maximizes its utility. Countries share the same state of pollution, and benefit from their own as well as from the other player's abatement policies. In the model, emissions are exogenous and countries choose their abatement effort, which can be interpreted as countries choosing net emissions. Unlike the existing literature, which focuses on *homogeneous* strategies, here, the players are not confined to adopt the same strategy space. Rather, we introduce the possibility of *heterogeneous* strategies, i. e., a game in which one country commits to a stream of abatement efforts while the other one decides upon its abatement at each moment in time.[3]

The strategy of the open-loop player, however, differs from the classical case where both players adopt open-loop strategies. Rather than being passive and simply taking as given the other player's strategy, here, the open-loop player adapts her strategy, at the beginning of the game, to the possible updating the rival player may take. We refer to this as an *anticipating open-loop strategy*. For an

[3]Our heterogeneous strategy setting should be distinguished from an asymmetric setting where both players still adopt the same strategy space (e.g. Markovian) but do not take the same action. For example, Reinganum (1981) develops a model, where two identical players play *asymmetric* Markovian strategies.

infinite time horizon, this type of heterogeneous strategies may admit a subgame-perfect nondegenerate Markovian Nash Equilibrium. The great advantage of this strategy is that, in addition to the stationary path, it allows us to study short-run trajectories, which usually are not subgame perfect.[4]

Previous research that considers the effects of *unilateral emission reductions*, defined as the emission level below what the best response in Nash equilibrium would commend, has been scarce and most of it adopts a static approach. One of the first studies analyzing the effects of unilateral emissions reduction on total emissions in a static environmental game is Hoel (1991). The author finds that unilateral reduction of emissions may in fact lead to an increase in total emissions and also reduce total global welfare. The precise effect depends on whether the setting of the game is non-cooperative or cooperative; and if cooperative, on whether an agreement is reached or not. Other static approaches are Fankhauser and Kverndokk (1996), Pereau and Tazdait (2001) and Brechet et al. (2008). In contrast, Zagonari (1998) compares the long-run effect of unilateral initiatives on the level of pollution in a static game and on the stock of pollution in a dynamic game. He finds that in the static framework coordinating pollution strategies always leads to a lower level of pollution than unilateral initiatives. In contrast, when the setting of the game is dynamic, appropriate unilateral initiatives may lead to a stock of pollution lower than in the coordinating equilibrium. However, in his model unilateral initiatives merely refer to the ability of environmental groups to change the representative agent's preferences toward either consumption, the environment or the future generations. The game continues to be played in homogeneous strategies where both players follow Markovian strategies and conditions on the aforementioned preference parameters are established such that the stock of pollution is lower than under a coordination of strategies.

The present study contributes to and extends the previous literature by introducing a *dynamic* games approach with *heterogeneous* strategies to address the consequences of *unilateral commitment*. Given that climate outcomes are determined by global emissions, from the point of view of any single country, efforts to reduce emissions unilaterally appear a waste of resources. On the one hand, the well-known 'tragedy of the commons' result predicts that the welfare of citizens of a unilaterally acting country should worsen. On the other hand, following unilateral efforts to cut emissions, total emissions might, nevertheless, be higher compared to a world in which both countries play Markovian strategies.

The introduction of heterogeneous strategies (meaning that the two players choose actions from different strategy spaces) is an attempt to better map the recent announcements of unilateral measures to reduce emissions into a theoretical model. To study the consequences of unilateral commitment we start from the premise that countries have agreed on a common long-term valuation of environmental quality (i.e. they have a common target of environmental value). The Copenhagen Accord, negotiated at the Copenhagen Climate Conference in 2009, formulates the need to limit global temperature increase

[4]More details on our heterogeneous strategy setting are provided in Section 3.

to 2 degrees Celsius. The political leaders of the participating states agreed "...that deep *cuts in global emissions* are required according to science ... so as to hold the increase in global temperature below 2 degrees Celsius...".[5] One may interpret this as a public recognition of the threat of negative consequences of a global temperature increase above 2 degrees Celsius and a convergence toward a common target to which global warming should be limited. Even though needed, a joint strategy for tackling the problem of global warming has yet to be agreed on by the international community. However, as noted above, some countries have committed to unilateral cuts in emissions. The fact that an agreement on a pair of strategies on emission levels between (two) countries has not been observed has been a justification for analyzing dynamic games in Markovian strategies. With the introduction of heterogeneous strategies we wish to take into account that *unilateral* action might be taken to contribute to a common target value of environmental quality. In practice, we allow for the possibility of one country adopting an open-loop strategy and the other one a Markovian strategy as regards pollution abatement.

Given an agreement on the value of environmental quality, it seems natural to ask the question whether unilateral commitment can lead to a lower global stock of pollution than a common agreement among all countries? Or will the country that unilaterally commits only make itself worse off? Also, how will unilateral commitment affect the aggregate level of utility?

Our main results can be summarized as follows. Not surprisingly, we find that the standard free-riding result holds: the country adopting a feedback strategy (the non-committed player) will always free-ride on the country adopting an open-loop strategy (the unilaterally committed player). The committed player will always be worse-off in terms of utility, because he abates more than under homogeneous strategies to compensate for the free-riding attitude of the non-committed player. Moreover, a Nash equilibrium in heterogeneous strategies will also be inferior in terms of aggregate utility compared to an equilibrium in homogeneous strategies.

Our major finding is that accumulated pollution under heterogeneous strategies may, however, be lower than under homogeneous strategies. This will depend on how much they value environmental quality. A sufficiently high value attributed to environmental quality can induce an abatement effort by the committed player that more than compensates for the reduced effort of the non-committed player and that leads to less accumulated pollution than under homogeneous strategies. We interpret this latter result as a signal that a comprehensive and homogeneous environmental agreement may not necessarily always be the most desirable outcome in terms of pollution reduction. A situation where countries taking different paths may well lead to a better environmental outcome and represents an option which should not automatically be rejected as inferior.

[5] Copenhagen Accord (2009), p.2, http://unfccc.int/resource/docs/2009/cop15/eng/l07.pdf

The rest of the paper is organized as follows. Section 2 briefly describes a stylized differential game with transboundary pollution abatement to which it is possible to apply a strategy space with unilateral commitment. Section 3 presents the solution of the game under the heterogeneous strategy space, while Section 4 resorts to numerical simulations to compare the resulting time paths for the variables of interest under the various strategy combinations. Conclusions are provided in Section 5.

2 Model of transboundary pollution

Suppose there are two countries: i and j. Both produce consumption goods with pollution as a byproduct. These two countries undergo the same pollution state, $x(t)$, which evolves as follows[6]

$$\dot{x}(t) = E(t) - (u_i + u_j)\sqrt{x(t)} - \delta x(t), \ t \geq 0, \tag{1}$$

where the initial condition $x(0)$ is a given positive constant, and parameter $\delta \in [0,1]$ measures the pollution absorption rate of nature. $E(t) = E_i(t) + E_j(t)$ is a known positive function of pollution emissions. u_i and u_j are abatement rates of countries i and j, respectively. Furthermore, if emissions are larger than the natural rate of absorption of nature itself, pollution will increase without bound. Expression $-(u_i + u_j)\sqrt{x(t)}$ implies that the higher the pollution state, the more efficient is one unit of abatement. The square root term is used for analytical ease.

Facing the pollution-abatement problem, the two countries need to choose their abatement rate u_l, $l = i, j$, to maximize their utility

$$\max_{u_l} \int_0^{\overline{T}} e^{-r_l t} \left(-x(t) - \frac{\alpha_l}{2} u_l^2 \right) dt + S(x(\overline{T})), \ l = i, j, \tag{2}$$

subject to the state constraint (1), where $r_l \in [0,1)$ is the time preference parameter, α_l is a positive, constant adjustment cost coefficient. Furthermore, $x(\overline{T})$ is the final target of the pollution state of the world, where both players agree on a final date T, and the last term, $S(x(T))$, represents the utility from the state of the environment at this final date. We consider $\overline{T} \leq \infty$, where $S(x(\infty)) = 0$ and $S(x(T))$ is a given positive function in finite time $\overline{T} = T$, with $S_x < 0$.

In the next section, we study heterogeneous strategic solutions of this differential game.

[6]If we take into account uncertainty, this law of motion of pollution accumulation could be presented by an Ito stochastic differential equation and the objective function would change to expected utility.

3 Solving the game via Heterogeneous strategies

In this section, we focus on the solution of the game in a situation in which countries follow *heterogeneous strategies*. We provide the solutions under *homogeneous strategies* in the Appendix (That is most of the case in this part of literature, see the complete survey Long (2010). Therein, we present the case where both countries adopt a Markovian strategy, i. e., at each period of time, both countries decide on their abatement efforts according to the state of the world.[7] We also solve the game for the case where both countries adopt an open-loop strategy, i. e., at the beginning of the game, both countries commit to abatement levels independently of the state of the world at each moment in time. Rather than presenting the classical open-loop strategy for reason of comparison, here, it represents the first step in constructing the *anticipating open-loop strategy*,[8] the strategy that will be played by the committed player when countries follow *heterogeneous strategies*.

In the following we consider the case in which country j plays a Markovian strategy so that its abatement intensity changes over the state of pollution, $u_j = u_j(x,t)$. Country i, on the other hand, follows an anticipating open-loop strategy and commits at the beginning of the game by taking into account player j's Markovian strategy. Rather than being passive and simply taking as given country j's Markovian strategy, as in the classical case of homogeneous open-loop strategies, country i conjectures the possible updating of country j at the beginning of the game.

Since explicit trajectory solutions cannot be obtained, we use numerical simulations in Section 4 to attain time paths of the variables of interest allowing us to compare the outcomes of heterogeneous and homogeneous strategies.

We define a Heterogeneous Strategy Nash Equilibrium as follows:

Definition 1 *A 2-tuple* (Ψ_i, Ψ_j) *of functions* $\Psi_i : [0,+\infty) \to \mathbb{R}_+$ *and* $\Psi_j : [0,+\infty) \times [0,+\infty) \to \mathbb{R}_+$*, with* $\Psi_i = \Psi_i(t), \forall t \in [0,+\infty)$ *and* $\Psi_j = \Psi_j(x,t), \forall (x,t) \in \mathbb{R}^2_+$*, is called a Heterogeneous Strategic Nash Equilibrium if, for each* i, j*, an optimal control path* $u_i(\cdot)$ *of player* i *exists and is given by the open-loop strategy:* $u_i(t) = \Psi_i(t)$ *and an optimal control path* $u_j(\cdot)$ *of player* j *exists and is given by the Markovian strategy:* $u_j(t) = \Psi_j(x(t),t), \forall t \geq 0$.

This kind of Heterogeneous Strategy Nash Equilibrium was first introduced in Dockner et al (2000, p.87-92, Example 4.1) and further developed in Zou (2014). As emphasized in the latter, the ability to find

[7] For a detailed definition consult Dockner et al (2000) or Reinganum and Stokey (1985).

[8] The meaning of the *anticipating open-loop strategy* will become clear later. A precise definition may be found in Zou (2014).

the Nash Equilibrium in heterogeneous strategies crucially depends on how the commited player guesses a rival's Markovian strategy. [9] Suppose that player i, who adopts an open-loop strategy, *conjectures* its rival's Markovian strategy $\Psi_j(x,t)$ as given, and hence, faces the following optimization problem:

$$\begin{cases} \max_{u_i} \int_0^{\overline{T}} e^{-r_i t} \left[-x(t) - \frac{\alpha_i}{2} u_i^2(t) \right] dt + S_i(x(\overline{T})), \\ \text{subject to } \dot{x} = E(t) - (u_i(t) + \Psi_j(x,t))\sqrt{x} - \delta x, \end{cases} \tag{3}$$

Then, player i's Hamiltonian function is

$$H_{m,i}(x,u_i,\lambda_{m,i},\Psi_j(x,t),t) = \left(-x(t) - \frac{\alpha_i}{2} u(t)_i^2\right) + \lambda_{m,i}(t)\left[E(t) - (u_i(t) + \Psi_j(x,t))\sqrt{x(t)} - \delta x\right],$$

where $\lambda_{m,i}$ is player i's costate variable. Pontryagin's maximum principle leads to

$$u_i(t) = -\frac{\lambda_{m,i}(t)\sqrt{x(t)}}{\alpha_i} \tag{4}$$

and

$$\dot{\lambda}_{m,i}(t) = r_i\lambda_{m,i}(t) - \frac{dH_{m,i}}{dx} = (r_i+\delta)\lambda_{m,i}(t) + 1 + \frac{u_i(t)+\Psi_j(x,t)}{2\sqrt{x(t)}}\lambda_{m,i}(t) + \lambda_{m,i}(t)\sqrt{x(t)}\frac{\partial \Psi_j(x,t)}{\partial x}. \tag{5}$$

Player i conjectures player j's optimal strategy $\Psi_j(x,t)$ as $\Psi_j(x,t) = -\frac{\lambda_{m,j}(t)\sqrt{x}}{\alpha_j}$ for all $(x,t) \in \mathbb{R}_+^2$ with $\lambda_{m,j}$ being the costate variable of player j. The precise way in which player i conjectures the functional form of player j' s optimal strategy, $\Psi_j(x,t)$, derives from the solution to the game played in homogeneous open-loop strategies, thus based on Pontryagin Maximum principle. However, here, the open-loop player anticipates the rival's Markovian strategy and incorporates it into her own strategic plan. There are two terms in equation (5) that distinguish the strategy of the anticipating open-loop player in the present heterogeneous strategies game from the classical open-loop strategy. First, notice that the open-loop player conjectures the rival's strategy $\Psi_j(x,t)$ to be dependent on the state of pollution. Second, the anticipating open-loop player anticipates how her rival's strategy depending on the state of pollution. The presence of the last term in equation (5) captures this important detail.

With the above conjecture at hand, the costate variable of player i may be written as

$$\dot{\lambda}_{m,i}(t) = 1 + (r_i+\delta)\lambda_{m,i} - \frac{\lambda_{m,j}}{\alpha_j}\lambda_{m,i} - \frac{\lambda_{m,i}^2}{2\alpha_i}, \tag{6}$$

with terminal (or transversality) condition $\lambda_{m,i}(\overline{T}) = \frac{\partial S_i(x^*(\overline{T}))}{\partial x}$.

[9] Guessing techniques are used very often in the application of differential games. See more in Long (2010).

Remark. The transversality condition of costate variable $\lambda_{m,i}$ at final date, $\overline{T}$, depends on the optimal value of the state variable, $x^*(\overline{T})$.

It is easy to show that player i's Hamiltonian function is linear in the state variable $x(t)$ and hence concave. Therefore, the above first order conditions give not only necessary but also sufficient conditions for player i's maximization problem.[10] Thus, the above choice of $u_i(t)$ is an open-loop strategy for player i.

Player j, who adopts a Markovian strategy, faces the maximization programme

$$\begin{cases} \max_{u_j} \int_0^{\overline{T}} e^{-r_j t} \left[-x(t) - \frac{\alpha_j}{2} u_j^2(t)\right] dt + S_j(x(\overline{T})), \\ \text{subject to } \dot{x} = E(t) - (\Psi_i(t) + u_j(t))\sqrt{x(t)} - \delta x(t). \end{cases} \tag{7}$$

Notice that the unilateral commitment of player i is taken into account in that her strategy, Ψ_i, is state independent. We may, thus, write player j's Hamiltonian function as

$$H_{m,j}(x, u_j, \lambda_{m,j}\Psi_i(t), t) = \left(-x(t) - \frac{\alpha_j}{2} u_j^2\right) + \lambda_{m,j}\left[E(t) - (u_j + \Psi_i(t))\sqrt{x(t)} - \delta x\right],$$

and Pontryagin's Maximum principle leads to

$$u_j(t) = -\frac{\lambda_{m,j}(t)\sqrt{x(t)}}{\alpha_j}, \tag{8}$$

and

$$\dot{\lambda}_{m,j}(t) = 1 + (r_j + \delta)\lambda_{m,j} + \frac{u_j + \Psi_i(t)}{2\sqrt{x}}\lambda_{m,j},$$

with $\Psi_i(t) = u_i^*(t) = -\frac{\lambda_{m,i}(t)\sqrt{x(t)}}{\alpha_i}$, as player i plays open loop strategy. It is straighforward to check that the costate variable of player j satisfies

$$\dot{\lambda}_{m,j}(t) = 1 + (r_j + \delta)\lambda_{m,j} - \frac{\lambda_{m,i}}{2\alpha_i}\lambda_{m,j} - \frac{\lambda_{m,j}^2}{2\alpha_j} \tag{9}$$

with terminal condition $\lambda_{m,j}(\overline{T}) = \dfrac{\partial S_j(x^*(\overline{T}))}{\partial x}$.

Again, the Hamiltonian function of player j is linear in the state variable $x(t)$ and hence, concave. Therefore, the above first order conditions indeed correspond to a Markovian strategy of player j.

Combining the above results, the state equation reads

$$\dot{x} = E(t) + \left(\frac{\lambda_{m,i}}{\alpha_i} + \frac{\lambda_{m,j}}{\alpha_j}\right)x - \delta x. \tag{10}$$

[10]Sufficiency can be found, for example, in Dockner et al (2000; Theorem 3.2)).

Mathematically, solving the nonlinear dynamic system given by (6), (9), and (10), with an initial condition on the state variable and transversality conditions on the costate variables, yields the solution of the dynamic game. Substituting this solution into (4) and (8) yields the Heterogeneous Strategic Nash Equilibrium.[11]

The specification of the terminal conditions on the costate variables plays an important role in the solution of the game. To fix the terminal conditions, in our solution, we will assume that both players share a common value for environmental quality. This assumption is based on an interpretation of the widespread recognition of the need to limit global warming below 2 degrees Celsius to implicitly reveal a common value for environmental quality.

We "translate" this assumption into our theoretical framework by imposing that both players share the same shadow value of the stock of pollution at the final date $\overline{T}$. In other words, we impose that both players have an identical terminal condition given by

$$\lambda_i(\overline{T}) = \lambda_j(\overline{T}) = \frac{\partial S_j(x^*(\overline{T}))}{\partial x} = \frac{\partial S_i(x^*(\overline{T}))}{\partial x}.$$

To share a common value for environmental quality does not, however, necessarily imply to share the same action plan to preserve the environment, and, the heterogeneous strategy allows for a distinction between the choice of strategy and value for environmental quality. In other words, we postulate that it is well possible for countries to ascribe the same value for environmental quality, and yet choose different strategies. In light of the fact that some countries have committed unilaterally to reduce GHG emissions in trying to keep increases in global temperature below 2 degrees Celsius while others have opted not to commit, we believe our specification not to be too bold. In fact, considering these circumstances, studying the consequences of unilateral commitment under the premise that both players have agreed on a common value of environment appears to be a worthwhile endeavor.

Nevertheless, due to the nonlinearity, it is not easy to solve explicitly the above dynamic system. In the sequel, in order to allow for comparability with homogeneous strategies, identical assumptions on parameters will be imposed: $r_i = r_j = r$, $\alpha_i = \alpha_j = \alpha$, $\delta = 0$.

[11]It is straightforward to see that the latter is indeed a non-degenerate Markovian Nash Equilibrium.

The above differential game boils down to solving the following system

$$\begin{cases} \dot{x}(t) = E(t) + (\lambda_{m,i} + \lambda_{m,j})\ \dfrac{x}{\alpha}, \\ \dot{\lambda}_{m,i}(t) = 1 + r\lambda_{m,i} - \dfrac{\lambda_{m,j}}{\alpha}\lambda_{m,i} - \dfrac{\lambda_{m,i}^2}{2\alpha}, \\ \dot{\lambda}_{m,j}(t) = 1 + r\lambda_{m,j} - \dfrac{\lambda_{m,i}}{2\alpha}\lambda_{m,j} - \dfrac{\lambda_{m,j}^2}{2\alpha}, \end{cases} \tag{11}$$

given an initial value x_0, and the terminal condition $\lambda_{m,i}(\overline{T}) = \lambda_{m,j}(\overline{T}) = \frac{\partial S(x^*(\overline{T}))}{\partial x}$.

4 Numerical analysis

Because no explicit solutions of equation (11) can be obtained, we resort to simulations to compare homogenous and heterogeneous strategies and to illustrate their qualitative differences.

It should be stressed that the aim of our analysis is not to calibrate the model, but rather to present the qualitative patterns of the solution trajectories.

The game is numerically solved in two steps. Subsequently, we describe the two-step procedure for the case in which both play open-loop strategies.[12] In the first step, we make use of the fact that we know the final shadow value, $\lambda(T)$, to recursively retrieve the series of $\lambda(t)$ from period $t = T-1$ to period $t = 0$ for both countries using equation (14). In the second step, knowing the initial pollution level x_0, we can find the evolution of the pollution stock until date T through equation (1), using the solution for the abatement level obtained from equation (13). All other variables can be computed once we know the evolution of $\lambda(t)$ and $x(t)$.

The numerical analysis - setup. The numerical analysis illustrates the evolution of the pollution stock over a 20 year horizon (T in Table 1) in a hypothetical two-country world, where atmospheric carbon dioxide (CO_2) concentration is taken as a measure of the pollution stock. Its initial level (x_0 in Table 1), amounts to the current level of 400 parts per million (ppm). We impose a common shadow value of the stock of pollution 20 years after the initial date, $\lambda(T)$, which means that both countries value environmental quality equally (e.g. recognizing the need to limit global warming below 2 degrees Celsius). As stated above, the countries under consideration are symmetric in the sense that they share the same values for all parameters (see Table 1). In contrast to the existing literature, we allow for the

[12]The numerical solutions for the cases in which both play Markovian or heterogeneous strategies are obtained analogously.

case where they adopt different strategies, i.e. one playing an anticipating open-loop and the other one a Markovian strategy. In particular, in the following analysis, we focus on the evolution of individual abatement ($u_1\sqrt{x}, u_2\sqrt{x}$), total abatement ($(u_1+u_2)\sqrt{x}$), pollution stock (x), aggregate environmental cost ($-\lambda\Delta x$) as well as aggregate utility, considering three strategy combinations (both play open-loop, both adopt Markovian startegy and finally, the heterogeneous strategy case).

Table 1: Parameter values

Periods	T	20
Initial pollution stock	x_0	400
Time preference parameter	r_1, r_2	0.02
Natural absoprtion rate	δ	0.05
Emissions	E_1, E_2	2
Adjustment cost parameter	α_1, α_2	100

The numerical analysis - targets.

The central problem one encounters when solving numerically our game via the above mentioned procedure is that of fixing the terminal conditions[13]. While the initial condition imposed on stock of pollution can be easily fixed using the current CO_2 stock, $x_0 = 400$, to pin down $\lambda_{m,i}(\overline{T})$ and $\lambda_{m,j}(\overline{T})$ we need determine the utility for the pollution state at the final date, $S(x^*(\overline{T}))$, in the objective function (2), which is likewise not a straightforward task. Here, basing on the explicit solution (17), we truncate this solution at time $\overline{T}$ and apply to the short-run solution (18). Thus, we set the terminal state as $S(x(\overline{T})) = e^{(\frac{r\alpha}{2}-A)x(\overline{T})}$. The transversality condition for the shadow value λ is thus $\lambda(\overline{T}) = S_x(x^*(\overline{T})) = (\frac{r\alpha}{2}-A)e^{(\frac{r\alpha}{2}-A)x^*(\overline{T})}$, where $x^*(\overline{T})$ is the optimal solution at $\overline{T}$.

[13] We could impose other terminal condition along the lines of the target of the EU 2020 energy package, which is defined as a 20 per cent reduction in EU greenhouse gas emissions from 1990 levels. Economically, such a condition is not problematic, however, mathematically, it raises severe issues. Notice that this way of setting a terminal condition (See Goldman (1968), Chiang (2000) and references therein, for more and systematic statements of different types of transversality conditions.) by imposing both a terminal time T and a terminal stock, $x(T)$, implicitly imposes also $\lambda_{m,i}(T) = \lambda_{m,j}(T) = \frac{\partial S(x^*(\overline{T}))}{\partial x}$. With the initial condition, $x(0)$, being given at the same time, this lead to a rather more complicated terminal condition on the costate variables (See Theorem 8.2 and 4.1, Hartl et al (1995).). Thus, to avoid this difficulty and to be consistent with the theoretical part developed in the previous section, we choose to impose the terminal conditions on the costate variables, $\lambda_{m,i}(\overline{T})$ and $\lambda_{m,j}(\overline{T})$, in system (11).

Under the present parameter configuration the long-run steady-state value of λ is equal to $\lambda(+\infty) = (\frac{r\alpha}{2} - A) = -9$.[14] Since the long-run cannot be reached in finite time, in the following, we choose the transversality value $\lambda(\overline{T})$ close to, but higher than, its long-run steady-state value. More precisely, in the following simulations, we assume the target values are given and we present two different transversality conditions: (i) a *stringent target*, setting $\lambda(T) = -8.3$ close to its long-run solution, and (ii) a *loose target*, where $\lambda(T) = -8$ and less close to its long-run value.

Though the choice of a common target of environmental quality value, $\lambda(T)$, at the last period of the planning time is not at the heart of this study, we subscribe to the fact that the terminal condition is of crucial importance for policy makers. Appropriately choosing the terminal condition and setting the targets would however deserve a study on its own, and is beyond the aim of this paper.

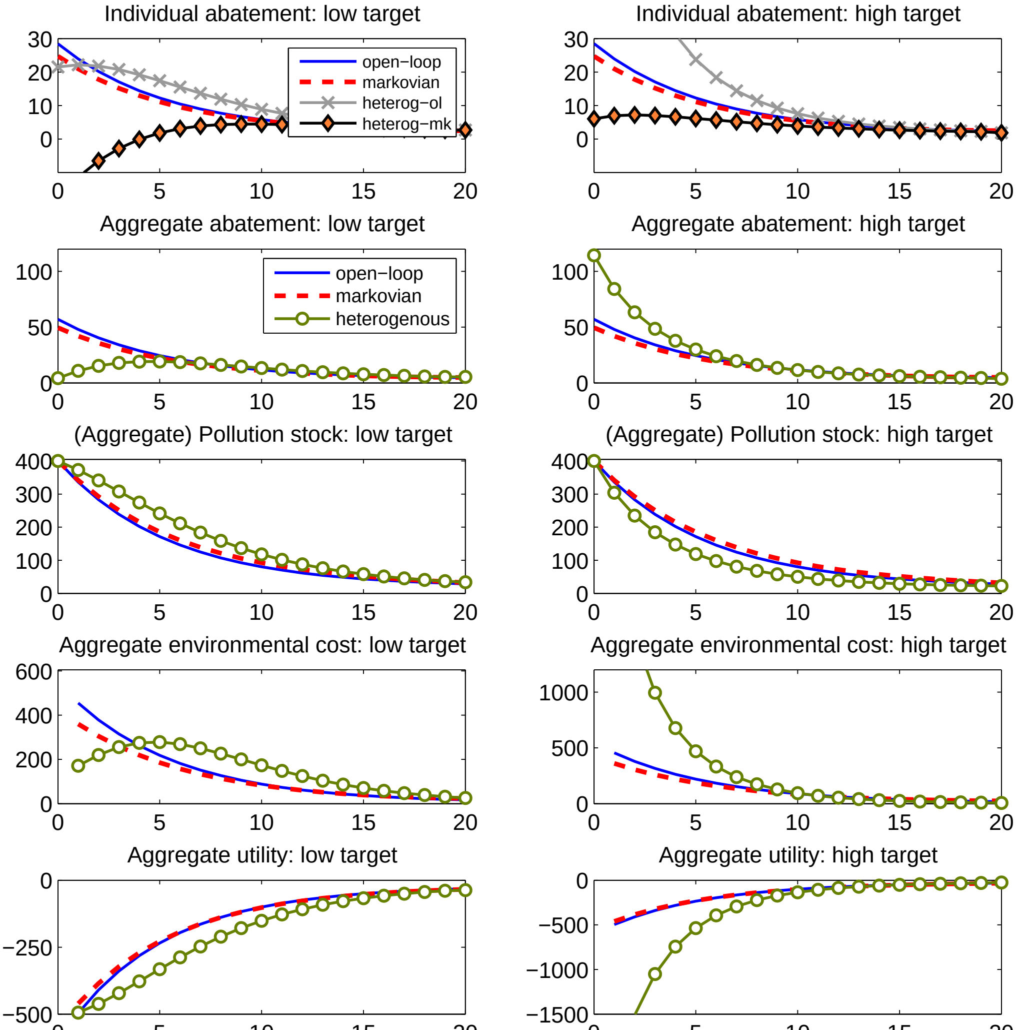
The numerical analysis - findings. Our main results under the different strategies are depicted in Figure 1: (i) "open-loop" stand for the case where both countries play open-loop strategies, (ii) "markovian" considers the case where both adopt Markovian strategies and (iii) "heterogeneous" represents the heterogeneous strategies case, where one country adopts an anticipating open-loop strategy ("heterog-ol") and the other one a Markovian strategy ("heterog-mk"). Results under the loose target are depicted on left panel graphs and results under the high stringent target on the right panel graphs. The two upper panels show individual results while the other eight graphs depict aggregate results for the two countries.

We notice that independently of the target, the individual abatement effort is lower when both countries adopt Markovian strategies (*dashed line*) than open-loop-strategies (*solid lines*). This result is also confirmed in heterogeneous strategies, where the Markovian strategy player (*line with diamonds*) freerides on the open-loop strategy player (*line with crosses*), and hence contributes less to abatement. In terms of aggregate results, the homogenous Markovian strategy case leads to less abatement and thus to more pollution than the homogenous open-loop strategy case, along the whole trajectory path, whether stringent or loose transversality conditions are applied (second and third row graphs).

However, the target matters when we look at heterogeneous strategies. When both players take a more lax stance towards the environment, $\lambda(T) = -8$, less effort will be devoted to abatement and the pollution stock will be higher than under homogenous strategies. However, in the more stringent target case, $\lambda(T) = -8.3$, which is closer to the long run optimal solution $\lambda(+\infty) = (\frac{r\alpha}{2} - A) = -9$, cumulated abatement is higher than in the other two strategies, which leads to a lower pollution stock. In other words, if both players highly weight environmental issues, it will lead both players to abate more in the aggregate, especially the open-loop player who commits right from the beginning. The

[14] Another possibility to choose this function would have consisted in taking the same function as in the utility function (2), that is, $S(x(\overline{T})) = -e^{-r\overline{T}}x(\overline{T})$. Though this is a straightforward choice from the utility function, it is hard to justify the long-run solution if $S(x(T)) \to 0$ as $T \to +\infty$.

Figure 1: Low target (left panel) versus high target (right panel) under different strategies



The figure shows the evolution of different variables over a 20-year horizon under various strategies. 'Markovian' and 'Open-loop' refer to the cases where both players adopt Markovian and open-loop strategies, respectively. In the heterogeneous strategy case, 'Heterog-ol' depicts the results of the player adopting an open-loop strategy and 'Heterog-mk' the results of the player adopting a Markovian strategy, while 'Heterogeneous' shows the aggregate results of the two countries.

tougher transversality condition leads the open-loop player to exert much more effort to clean up the environment. In fact, the country adopting open-loop strategy furnishes an abatement effort that more than compensates the small abatement effort of the Markovian strategy player, who abates less than in the homogenous strategy case.

Besides, total abatement costs (graphs 'aggregate environmental cost') are higher with heterogeneous strategies than with homogenous strategies. This is evident in the stringent target case. In the lax target case, the Markovian strategy player keeps freeriding, but at the same time less effort is devoted by the open-loop player, which explains the lower abatement cost in the initial period of the game. Nonetheless, towards the final period, more aggregate effort is unfold to meet the terminal target. Finally, the bottom graphs of Figure 1 display aggregate utility. Whether lax or stringent targets are applied, heterogeneous strategies always lead to lower aggregate utility. In fact, in the heterogeneous strategy case, the free-riding attitude of the Markovian makes the open-loop player suffer more than when the other player also commits (homogenous strategies). This result is even reinforced in case of tougher transversality conditions.

5 Conclusion

The present paper studies the case in which one GHG emitter unilaterally commits to reduce its GHG emissions while other major emitters, do not follow the same strategy. To address this issue, we rely on a dynamic game framework, where countries can adopt heterogeneous strategies, where one player adopts a feedback strategy and the other one anticipating open-loop strategy. This contribution being novel in the literature and is necessary to account for situations where unilateral commitment can take place, as in the case of the climate change negotiations. Our results show that under certain circumstances, it can be desirable from a global environmental point of view to have different groups of countries adopting heterogeneous strategies in terms of pollution mitigation. More particularly, we show that (i) in heterogeneous strategies, we may end up in a situation where the level of accumulated pollution is lower than in the open-loop Nash equilibrium case. This will happen if participants commit to a stringent enough target in terms of environmental quality, i.e. if both countries highly value the environment; (ii) in any case, the country adopting a feedback strategy will free ride more than the open-loop strategy player, independently on the set target; and finally, (iii) if the pollution abatement target is low, for both players there is a higher incentive to free ride, which will lead to higher levels of pollution than in the homogenous case.

Our analysis assumed that there is no uncertainty in terms of the environmental impact of GHG emissions. Future work however may integrate uncertainty, which is of tremendous importance when it comes to set policy prescription for outcomes which may occur in the very long term such as climate change. We envisage two alternative avenues to tackle uncertainty: (i) one possibility would be to make the final state of the world CO_2 level uncertain and work with expectations, although the state of the CO_2 accumulation and the effort of cleaning up would remain deterministic; (ii) another possibility would be to let CO_2 accumulation itself follow a stochastic process and the law of motion of CO_2 would be governed by a stochastic differential equation. Doing so would certainly enrich the analysis, and provide further insight into our understanding about how to cope with international climate change negotiations.

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A Appendix:Homogenous strategies

A.1 Open-loop strategies

For simplicity, we suppose the two countries are symmetric and commit to an abatement strategy based only upon time t, ignoring the current state of pollution. Player i's objective function consists in maximizing her utility stream with respect to $u_i(t)$

$$\max_{u_i(t)} \int_0^{\overline{T}} e^{-r_i t} \left(-x(t) - \frac{\alpha_i}{2} u_i^2 \right) dt + S(x(\overline{T})), \tag{12}$$

subject to the state constraint (1).

It is easy to see that player i's Hamiltonian function[15] is

$$H_i(x, u_i, \lambda_i, t) = \left(-x(t) - \frac{\alpha_i}{2} u_i^2 \right) + \lambda_i \left[E(t) - (u_i + u_j^*(t)) \sqrt{x(t)} - \delta x(t) \right],$$

where u_j^* represents the optimal strategy of variable u_j of player j.

Pontryagin's maximum principle leads to

$$u_i(t) = -\frac{\lambda_i(t)\sqrt{x(t)}}{\alpha_i}, \tag{13}$$

and the costate variable, the shadow value here, is

$$\dot{\lambda}_i(t) = r_i \lambda_i(t) - \frac{\partial H_i}{\partial x} = (r_i + \delta)\lambda_i + \frac{u_i + u_j^*}{2\sqrt{x}} \lambda_i + 1, \tag{14}$$

with transversality condition

$$\lim_{t \to \infty} e^{-r_i t} \lambda_i(t) x(t) = 0, \text{ or } \lambda(T) = S_x(x^*(T)). \tag{15}$$

Remark. From (14), we can see that the natural absorption rate δ has the same effect as the time preference r_i. In the following calculation, we will set $\delta = 0$, i.e. set r_i as $r_i + \delta$, while refereing to it when interpreting our results.

Furthermore, supposing symmetry between countries, i.e. $r_i = r_j = r$, $\alpha_i = \alpha_j = \alpha$, we have

$$u_i(t) = u_j(t) = u(t), \; \lambda_i(t) = \lambda_j(t) = \lambda(t),$$

[15] Notice there is an exogenous function $E(t)$ which may depend on time t, in particular during the short run. The dynamic programming HJB equation is a real partial differential equation with term V_t, given there is no boundary (or transversality) condition and non-linear-quadratic setting. In general, this makes the search for a solution more difficult, and stationarity usually does not hold any more. From this point of view, with $E(t)$ a given function of time t, Pontryagin's maximum principle provides richer results.

This symmetry is preserved for the optimal values, such that the above analysis yields the following system

$$\begin{cases} u^*(t) = -\frac{\lambda(t)\sqrt{x(t)}}{\alpha}, \\ \dot{x} = E(t) + \frac{2\lambda x(t)}{\alpha} - \delta x(t), \\ \dot{\lambda} = 1 + r\lambda - \frac{\lambda^2}{\alpha}, \end{cases} \tag{16}$$

with the initial condition, and the transversality condition (15). The shadow price λ (which is a *Riccati equation*) can be solved explicitly, while pollution state x is a linear equation. Hence, we obtain the explicit solution for the open loop problem. For the long run steady state, it yields,

$$\begin{cases} \lambda(t) & = \overline{\lambda} = \frac{\alpha r}{2} - A(<0), \ \overline{T} = \infty, \\ x_o(t) & = e^{\frac{2\overline{\lambda}}{\alpha}t}\left[x(0) + \int_0^t e^{-\frac{2\overline{\lambda}}{\alpha}s}E(s)ds\right], \ \overline{T} = \infty \end{cases} \tag{17}$$

with $A = \sqrt{\alpha + \left(\frac{r\alpha}{2}\right)^2}$.

While, if $\overline{T} = T < \infty$, the short run dynamics are given as follows: for $0 < t < T$,

$$\begin{cases} \lambda(t) = \begin{cases} \frac{r\alpha}{2} + A - \frac{2A}{C_o e^{\frac{2At}{\alpha}} + 1}, \text{ if } S_x(x^*(T)) \neq \frac{r\alpha}{2} - A, \\ \frac{r\alpha}{2} - A(=\overline{\lambda}), \text{ if } S_x(x^*(T)) = \frac{r\alpha}{2} - A, \end{cases} \\ x_o(t) = e^{\int_0^t \frac{2\lambda(s)}{\alpha}ds}\left[x(0) + \int_0^t e^{-\int_0^\tau \frac{2\lambda(s)}{\alpha}ds}E(\tau)d\tau\right], \ \overline{T} = T < \infty, \end{cases} \tag{18}$$

with $C_o = \frac{A + \left[S_x(x^*(T)) - \frac{r\alpha}{2}\right]}{A - \left[S_x(x^*(T)) - \frac{r\alpha}{2}\right]} e^{-\frac{2A}{\alpha}T}$.

The above explicit solutions provide the short run dynamics of the system under open-loop strategy. Comparing with infinite time horizon, where the shadow price is always constant over time, if there is finite time target for the pollution level, $S(x(T))$, the shadow value, $\lambda(t)$, changes over time to match this final target. The explicit solution yields

$$\dot{\lambda}(t) = \begin{cases} > 0, \text{ if } 0 > S_x(x^*(T)) > \frac{r\alpha}{2} - A, \\ 0, \text{ if } S_x(x^*(T)) = \frac{r\alpha}{2} - A, \\ < 0, \ \text{ if } S_x(x^*(T)) < \frac{r\alpha}{2} - A(<0), \end{cases}$$

Recall the shadow value λ measures the unit effect of pollution, hence, it is not surprising to notice that the short run dynamics depend essentially on the slope of the terminal condition S_x, rather than on the terminal condition itself. Except in a perfect setting where the dynamics are constant and equal to exactly the long run steady state, we have either over-shooting by imposing a too fast change at the end ($\mid S_x(x^*(T)) \mid$ is too large, $S_x(x^*(T)) < \frac{r\alpha}{2} - A$) or under-shooting by imposing a too slow change at the end ($\mid S_x(x^*(T)) \mid$ is too small, $\frac{r\alpha}{2} - A < S_x(x^*(T)) < 0$). In any of these two cases, the shadow price will not converge to its long run steady state. Therefore, the short run target may mislead the shadow value function during the transition, unless it is already at its steady state level.

We can conclude the above analysis as follows.

Proposition 1 *The optimal open-loop strategies yield:*

(I) In the short-run, the dynamics of the shadow value and the level of pollution depend on the terminal condition and the slope of the terminal condition, and are given by (18).

(II) In the long run, the shadow value is constant and the level of pollution is changing over time, as noted in (17)

Moreover, there are three possibilities based on the emission of pollution.

(II.1) If pollution emission $E = \overline{E}$ is a constant, there is a unique steady state, where states of pollution and abatement are constants and given by

$$x_o^* = -\frac{\alpha \overline{E}}{2\lambda^*}, \; u_o^* = -\frac{\lambda^* \sqrt{x^*}}{\alpha}.$$

(II.2) If pollution emissions $E = E(t)$ are increasing (or decreasing) over time with constant growth rate g_E, the pollution state will increase (or decrease) at the same rate g_E while abatement increases (or decreases) at rate $\frac{g_E}{2}$. But the shadow value of the pollution is the same constant as above.

(II.3) If pollution emission $E = E(t)$ is neither of the above cases, then there is no balanced growth path nor a steady state.

A.2 Markovian Nash Equilibrium

Now suppose that both countries can change the abatement strategies basing on the pollution state. Then each player is facing the same problem as above by choosing the abatement strategy $u_i(t) = \Psi_i(x(t),t)$ for a given guessing optimal strategy of player j, $\Phi_j(x,t), \forall (x,t) \in \mathbb{R}_+^2$. In the sequel, we will employ the similar conjecturing methods as the mixed strategy to look at one particular symmetric Markovian Nash Equilibrium.

Player i's Hamiltonian function is

$$H_i(x,u_i,\phi_i,\Phi_j(x,t),t) = \left(-x(t) - \frac{\alpha_i}{2}u_i^2\right) + \phi_i\left[E(t) - (u_i + \Phi_j(x,t)\sqrt{x(t)}\right],$$

with ϕ_i player i's costate variable and $\Phi_j(x,t)$ guessed player j's optimal strategy.

Pontryagin's maximum principle shows that

$$u_i(t) = -\frac{\phi_i(t)\sqrt{x(t)}}{\alpha_i}, \tag{19}$$

and

$$\dot{\phi}_i(t) = r_i\phi_i(t) - \frac{\partial H_i}{\partial x} = r_i\phi_i + 1 + \frac{u_i + \Phi_j(x,t)}{2\sqrt{x}}\phi_i + \phi_i\sqrt{x}\frac{\partial \Phi_j(x,t)}{\partial x}, \tag{20}$$

and its terminal condition. By conjecturing the optimal strategy of player j, $\Phi_j(x,t) = -\frac{\phi_j(t)\sqrt{x}}{\alpha_j}$ and with symmetric assumption and searching for symmetric solution, it follows, the optimal strategy and costate variable check

$$\begin{cases} u_i^*(t) = u_j^*(t) = u^*(t) = -\frac{\phi(t)\sqrt{x}}{\alpha}, \\ \dot{x} = E(t) + \frac{2\phi x}{\alpha}, \\ \dot{\phi} = 1 + r\phi - \frac{3\phi^2}{2\alpha}, \end{cases} \tag{21}$$

with initial condition and transversality condition.

Using similar methods to obtaining (18), we could get the similar explicit solutions for the short-run and long-run dynamics, especially the effect of the short-run target effect on the dynamics.

The long-run explicit solution to the Markovian strategy is given by

$$\phi(t) = \overline{\phi} = \frac{1}{3}\left(\alpha r - \sqrt{(\alpha r)^2 + 6\alpha}\right)(< 0), \quad x_m(t) = e^{\frac{2\overline{\phi}}{\alpha}t}\left[x(0) + \int_0^t e^{-\frac{2\overline{\phi}}{\alpha}s}E(s)ds\right]. \tag{22}$$

And the short run dynamics basing on the terminal condition are

$$\begin{cases} \phi(t) = \begin{cases} \dfrac{r\alpha}{3} + B - \dfrac{2B}{C_m e^{\frac{3Bt}{\alpha}} + 1}, \text{ if } S_x(x^*(T)) \neq \dfrac{r\alpha}{3} - B, \\ \dfrac{r\alpha}{3} - B(= \overline{\lambda}), \text{ if } S_x(x^*(T)) = \dfrac{r\alpha}{3} - B, \end{cases} \\ x_o(t) = e^{\int_0^t \frac{2\phi(s)}{\alpha}ds}\left[x(0) + \displaystyle\int_0^t e^{-\int_0^\tau \frac{2\phi(s)}{\alpha}ds}E(\tau)d\tau\right], \ \overline{T} = T < \infty, \end{cases} \tag{23}$$

with $C_m = \dfrac{B + \left[S_x(x^*(T)) - \frac{r\alpha}{3}\right]}{B - \left[S_x(x^*(T)) - \frac{r\alpha}{3}\right]}\, e^{-\frac{3B}{\alpha}T}$ and $B = \sqrt{\left(\frac{r\alpha}{3}\right)^2 + \frac{2\alpha}{3}}$.

That finishes the proof.