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Risky Rents

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Abstract

We consider a strategic contest game in which risk averse agents exert efforts to increase their share of a risky rent. We show that a unique symmetric equilibrium always exists under constant or decreasing absolute risk aversion. We derive conditions so that agents exert less efforts when they become more risk averse or when the rent is more risky. We also provide a case with convex impact functions in which an increase in risk may lead to an increase in rent seeking efforts.

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1 Introduction

Many economic situations can be modelled by contests in which agents compete for a prize or a rent.[1] Contests naturally vary by the degree of divisibility and of riskiness of the rent. The example of political lobbying is a good illustration of this variability. Consider firms exerting efforts in lobbying to influence political decisions. In some situations, the winner takes all, for instance when the effort is to obtain a monopoly position. In other situations however, for instance when firms in different sectors compete for subsidies, the benefit is typically shared among these competing firms.[2] Moreover, this benefit is often imperfectly known, for instance because it depends on future economic conditions.[3]

Motivated by these examples, we analyze in this paper the arguably realistic case of a game with a divisible and risky rent contested by risk averse agents. Some papers have already examined the effect of risk aversion in contests (e.g., Hillman and Katz 1984, Skaperdas and Gan 1995, Konrad and Schlesinger 1997, Treich 2010, Cornes and Hartley 2003 and 2012). However, in all these papers, the risk stems from the probabilistic nature of the contest, not from the rent itself. Indeed, agents compete to increase their respective winning probabilities of the rent. In other words, the rent is riskless, but the rent allocation process is risky. In the present paper, by contrast, agents compete to increase their respective rent shares. In our model, the risk comes only from the risky nature of the rent, not from the rent allocation process.[4]

To the best of our knowledge, Long and Vousden (1987) is the only paper analyzing a contest in which risk averse agents compete for a share of a divisible rent. However, their analysis significantly differs from ours, for two main reasons. First, they do not consider a risky rent, but instead a "risky

[1] Surveys and collections of seminal papers on contests include Nitzan (1994), Garfinkel and Skaperdas (2007), Corchón (2007), Konrad (2009), Long (2013), and Congleton and Hillman (2015)..

[2] Long and Vousden (1987), Wärneryd (1998) and Inderst, Müller and Wärneryd (2007) provide several examples of contests in which the rent is shared, including lobbying for trade protection or the allocation of a rent among the members of a cartel.

[3] As another example, consider an oligopoly game in which firms compete for market shares. This can be modelled as a strategic contest with a divisible rent (Menezes and Quiggin 2010). In this game, the "size" of the market may be risky, for instance because the demand curve is imperfectly known.

[4] A few papers (Harstad 1995, Warneryd 2003) consider a risky rent but assume risk neutral agents and instead focus on asymmetric information.

share" of the rent. In this sense, they still consider a risky rent allocation process similar to the rest of the literature. Second, Long and Vousden assume that the efforts are separable from the utility function (see, e.g., their equation (2)). This assumption essentially means that the effort and the rent are not commensurable.[5] This assumption is plausible in some situations, and technically convenient because it removes wealth effects. Nevertheless, this assumption is unusual in the literature since strategic contest models generally assume that both the rent and the efforts enter within the utility function of wealth of risk averse agents.

We develop a model based on the classical Tullock's rent seeking contest game (Tullock 1980). In our model, we assume that the rent is divisible and risky, and that the competing agents are risk averse. We examine the symmetric pure strategy Nash equilibrium of this game. In the standard case where the contest success function is determined by concave "impact functions" (Corchón 2007), a unique symmetric equilibrium always exists under constant absolute risk aversion (CARA) or decreasing absolute risk aversion (DARA). Then, we study the effects of risk and risk preferences on equilibrium efforts. Compared to a riskless situation, the introduction of risk on the value of the rent always leads to less rent seeking efforts. Moreover, agents always exert less efforts when they become more risk averse. The effect of more risk is more complex. We derive sufficient conditions under which risk averse agents exert less efforts when the rent becomes more risky. Nevertheless, we provide a case with convex impact functions in which an increase in risk may lead to an increase in rent seeking efforts.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 studies the equilibrium properties of the model. Section 4 examines the effects of increasing both risk aversion and risk on equilibrium efforts. Section 5 extends the analysis to convex impact functions. Section 6 concludes.

2 The Model

We consider a symmetric rent seeking contest in which agents compete for a share of a divisible rent. The rent $\widetilde{v}$ can be risky and satisfies the following

[5] Schroyen and Treich (2014) examine various types of contests which differ depending on whether the effort and/or the rent are commensurable. Also, Öncüler and Croson (2005) examine a contest with a risky rent, but make important separability assumptions.

properties.

A.1 $\widetilde{v}$ is a random variable with support contained in the interval $[\underline{v}, \overline{v}]$, where $0 < \underline{v} < \overline{v}$.

Let $E(\widetilde{v})$ denote the expected value of the risky rent. Agents can be risk averse with a common utility function u which satisfies the following properties.

A.2 u is thrice differentiable with $u' > 0$ and $u'' \leq 0$.

We assume that agents simultaneously exert costly efforts in order to increase their share of the rent. Let x_i denote agent i's expenditure, and let x_{-i} denote the strategy profile of all agents but i, with $i = 1, 2, ..., n$. Given a strategy profile (x_i, x_{-i}), the share of the rent awarded to agent i is given by the following contest success function (CSF):[6]

$$p_i(x_i, x_{-i}) = \frac{\phi(x_i)}{\sum_{j=1}^{n} \phi(x_j)}. \tag{1}$$

Following Corchón (2007), we interpret ϕ as the impact function of x_i in the contest, and we assume that ϕ satisfies the following properties.

A.3 ϕ is thrice differentiable with $\phi(0) = 0$ and $\phi' > 0$.[7]

Given a strategy profile $(x_i, x_{-i}) \neq (0, 0)$, the expected payoff of agent i is given by

$$U_i(x_i, x_{-i}) = E(u(p_i(x_i, x_{-i})\widetilde{v} - x_i)). \tag{2}$$

If $(x_i, x_{-i}) = (0, 0)$, we assume that the rent is shared equally, so that $U_i(0, 0) = Eu(\frac{\widetilde{v}}{n} - x_i)$.[8]

Definition *Let* $X_i = [0, \infty)$ *denote the set of feasible efforts of agent* i*. We define the "risky contest" described by* (1) *and* (2) *together with A.1, A.2 and A.3 as* $\Gamma = \{X_i, U_i, i = 1, ..., n\}$.

[6] Hillman and Rilley (1989), Fey (2008), Corchón and Dahm (2010) interpret CSFs as sharing rules. Corchón and Dahm (2010) also establish a connection between CSFs as sharing rules and bargaining. Note that both interpretations of CFSs, as sharing rules or as winning probabilities, are mathematically equivalent when agents are risk neutral.

[7] The functional form in (1) with the associated assumptions in A.3 is common in the literature on contests and was given an axiomatic foundation by Skaperdas (1996). A special case is the one in which $\phi(x) = x^m$, with $m > 0$, which was first introduced by Tullock (1980).

[8] An alternative assumption is that the rent is not distributed when $(x_i, x_{-i}) = (0, 0)$. This makes no difference to our results since this case cannot arise at the equilibrium (see section 3).

3 Equilibrium Properties

In this section, we examine the equilibrium properties of risky contests Γ, focusing on existence and uniqueness of a pure strategy equilibrium in the case where impact functions ϕ are concave.

Given the specification of the CSF in (1), agent i is assumed to solve the following maximization problem for a given $a = \sum_{j\neq i}^{n} \phi(x_j)$:

$$\underset{x_i}{Max}\ U_i = E(u(\frac{\phi(x_i)}{\phi(x_i)+a}\widetilde{v} - x_i)), \quad i = 1, 2, ..., n.$$

The first order conditions for a solution to this problem are:

$$E((\frac{\phi'(x_i)a}{(\phi(x_i)+a)^2}\widetilde{v} - 1)u'(\frac{\phi(x_i)}{\phi(x_i)+a}\widetilde{v} - x_i)) = 0, \quad i = 1, 2, ..., n. \tag{3}$$

The second order conditions

$$E((\frac{\phi''(x_i)\left(\phi(x_i)+a\right)a - 2(\phi'(x_i))^2 a}{(\phi(x_i)+a)^3}\widetilde{v})u'(\frac{\phi(x_i)}{\phi(x_i)+a}\widetilde{v} - x_i))$$

$$+E((\frac{\phi'(x_i)a}{(\phi(x_i)+a)^2}\widetilde{v} - 1)^2 u''(\frac{\phi(x_i)}{\phi(x_i)+a}\widetilde{v} - x_i)) < 0, \quad i = 1, 2, ..., n \tag{4}$$

are satisfied under ϕ concave. This implies that the solution of the n agents' first order conditions in (3) represents an equilibrium.

Given that the game is symmetric, we focus our attention on symmetric equilibria only. We first observe that $x_1 = x_2 = ... = x_n = 0$ is not an equilibrium. Indeed $x_i = 0$ is not the best response to $x_{-i} = 0$ since for any $x_i > 0$ small enough we have $Eu(\widetilde{v} - x_i) > Eu(\frac{\widetilde{v}}{n})$.[9]

We can then introduce our first result which is a general existence result.

Proposition 1 *There always exists an interior symmetric equilibrium in the risky contest Γ with ϕ concave.*

[9] More generally, if $x_{-i} = 0$, the expected utility of agent i approaches $Eu(\widetilde{v})$ as $x_i \to 0$. In other words, agent i's best response does not exist for $x_{-i} = 0$.

Proof: Let $f(x)$ denote any of the first-order conditions in (3) evaluated at the symmetric solution $x_1 = x_2 = ... = x_n = x$. Omitting the i-subscript, $f(x)$ can be written as follows:

$$f(x) = E((\Delta(x)\widetilde{v} - 1)u'(\frac{\widetilde{v}}{n} - x)) = 0, \tag{5}$$

where $\Delta(x) = \frac{n-1}{n^2}\frac{\phi'(x)}{\phi(x)}$. Observe that we have $f(0) = +\infty$ since $\Delta(0) = +\infty$. Moreover, observe that $\frac{\phi'(x)}{\phi(x)} \leq \frac{1}{x}$ under our assumptions on ϕ. As a result, $\Delta(x)$ can be made small enough, and thus $f(x)$ negative, for x large enough. This ensures that an interior symmetric solution to the n agents' first order conditions exists and that this solution characterizes an equilibrium since it is a global maximum.■

We can now state our second result which shows uniqueness under common assumptions on the utility functions.

Proposition 2 *The symmetric equilibrium in the risky contest* Γ *with* ϕ *concave is unique under* u *CARA or* u *DARA.*

Proof: The symmetric equilibrium is unique if the following single crossing property is satisfied: $f(x) = 0 \Longrightarrow f'(x) < 0$. We have

$$f'(x) = E((\Delta'(x)\widetilde{v})u'(\frac{\widetilde{v}}{n} - x)) - E((\Delta(x)\widetilde{v} - 1)u''(\frac{\widetilde{v}}{n} - x)).$$

Note that $E((\Delta'(x)\widetilde{v})u'(\frac{\widetilde{v}}{n} - x)) < 0$ since $\Delta'(x) < 0$ under ϕ concave. Thus, it is enough to show that $E((\Delta(x)\widetilde{v} - 1)u'(\frac{\widetilde{v}}{n} - x)) = 0$ implies $-E((\Delta(x)\widetilde{v} - 1)u''(\frac{\widetilde{v}}{n} - x)) \leq 0$.

Observe that $-E((\Delta(x)\widetilde{v} - 1)u''(\frac{\widetilde{v}}{n} - x)) = E((\Delta(x)\widetilde{v} - 1)u'(\frac{\widetilde{v}}{n} - x)A(\frac{\widetilde{v}}{n} - x))$ where $A(.) = -u''(.)/u'(.)$ is the degree of absolute risk aversion. Under CARA $A(.)$ is a constant so that $E((\Delta(x)\widetilde{v} - 1)u''(\frac{\widetilde{v}}{n} - x)) = 0$ and the sufficient condition for uniqueness is satisfied.

Under DARA, $A(\frac{v}{n} - x)$ is decreasing in v. Let us consider two cases. When $v \geq 1/\ \Delta(x)$, then $(\Delta(x)v - 1)u'(\frac{v}{n} - x)A(\frac{v}{n} - x) \leq (\Delta(x)v - 1)u'(\frac{v}{n} - x)A(\frac{1}{n\Delta(x)} - x)$. When $v \leq 1/\ \Delta(x)$, then we also have $(\Delta(x)v - 1)u'(\frac{v}{n} - x)A(\frac{v}{n} - x) \leq (\Delta(x)v - 1)u'(\frac{v}{n} - x)A(\frac{1}{n\Delta(x)} - x)$. Therefore we always have $E((\Delta(x)\widetilde{v} - 1)u'(\frac{\widetilde{v}}{n} - x)A(\frac{\widetilde{v}}{n} - x)) \leq E((\Delta(x)\widetilde{v} - 1)u'(\frac{\widetilde{v}}{n} - x)A(\frac{1}{n\Delta(x)} - x)) = 0$. Thus, we have $-E((\Delta(x)\widetilde{v} - 1)u''(\frac{\widetilde{v}}{n} - x)) \leq 0$ under DARA.■

4 Comparative Statics: The Effect of More Risk and More Risk Aversion

In this section, we examine the impact of increasing both risk aversion and risk on equilibrium efforts.

Note that if a symmetric equilibrium x^* exists, it is characterized by the following implicit solution:

$$\frac{\phi(x^*)}{\phi'(x^*)} = \frac{(n-1)}{n^2} \frac{E(\widetilde{v}u'(\frac{\widetilde{v}}{n} - x^*))}{E(u'(\frac{\widetilde{v}}{n} - x^*))}. \tag{6}$$

We first examine whether there is more effort compared to a situation where the rent is riskless (i.e., $\widetilde{v} =_d E\widetilde{v}$), or when the agents are risk neutral (i.e., $u'' = 0$).

Proposition 3 *Consider a risky contest Γ with ϕ concave and a symmetric equilibrium x^*. Let $\overline{x}$ be the unique equilibrium when the rent is riskless, or when the agents are risk neutral. Then, we always have $x^* \leq \overline{x}$.*

Proof: Under a riskless rent, or under risk neutrality, the symmetric equilibrium $\overline{x}$ is characterized by

$$E(\Delta(\overline{x})\widetilde{v} - 1) = 0.$$

Recall that $\Delta(x) = \frac{n-1}{n^2}\frac{\phi'(x)}{\phi(x)}$ is strictly decreasing in x under our assumptions on ϕ. This implies that the symmetric equilibrium $\overline{x}$ is unique.

We are done if we can show that $E(\Delta(x)\widetilde{v} - 1) = 0$ implies $f(x) = E((\Delta(x)\widetilde{v} - 1)u'(\frac{\widetilde{v}}{n} - x)) < 0$. Note that $f(x) = E(\Delta(x)\widetilde{v} - 1)Eu'(\frac{\widetilde{v}}{n} - x) + COV(\Delta(x)\widetilde{v} - 1, u'(\frac{\widetilde{v}}{n} - x))$. Under $E(\Delta(x)\widetilde{v} - 1) = 0$, we only need to show that the covariance term is negative. This is true since $\Delta(x)v - 1$ is increasing in v while $u'(\frac{v}{n} - x)$ is decreasing in v under risk aversion.■

Note that in the last Proposition no assumption is made about the uniqueness of the equilibrium. Indeed, since $\overline{x}$ is unique, the result is sufficient to show that all symmetric equilibria x^* are lower than $\overline{x}$.[10] We next obtain a general result about the effect of more risk aversion.

[10] In contrast, for the following results, we will assume that the symmetric equilibrium is unique. Note however that the results would still be informative even without this assumption. Indeed, one could still compare the lowest and highest equilibria of the game (Milgrom and Roberts 1994).

Proposition 4 *Consider a risky contest* Γ *in which a unique symmetric equilibrium exists. Then, more risk averse agents exert less efforts.*

Proof: Consider the implicit solution (6). Notice that the left hand side equals zero when $x^* = 0$. Moreover, the right hand side is strictly positive at $x^* = 0$, and thus crosses the left hand side from above. It is then enough to show that the term $\frac{E(\widetilde{v}u'(\frac{\widetilde{v}}{n}-x))}{Eu'(\frac{\widetilde{v}}{n}-x)}$ decreases with risk aversion to ensure that more risk aversion decreases x^*. As is usual in the comparative statics of risk aversion, we examine the effect of an increasing and concave transformation $T(.)$ of the utility function u (Pratt 1964). We thus want to show that

$$\frac{E[\widetilde{v}u'(\frac{\widetilde{v}}{n} - x)T'(u(\frac{\widetilde{v}}{n} - x))]}{E[u'(\frac{\widetilde{v}}{n} - x)T'(u(\frac{\widetilde{v}}{n} - x))]} \leq \frac{E(\widetilde{v}u'(\frac{\widetilde{v}}{n} - x))}{E(u'(\frac{\widetilde{v}}{n} - x))}.$$

We introduce the following probability density function

$$m(v) = \frac{d(v)u'(\frac{v}{n} - x)}{E(u'(\frac{\widetilde{v}}{n} - x))},$$

where $d(v)$ is the probability density function of the random variable $\widetilde{v}$. Then the previous inequality can be rewritten

$$\frac{\widehat{E}\widetilde{v}T'(u(\frac{\widetilde{v}}{n} - x))}{\widehat{E}T'(u(\frac{\widetilde{v}}{n} - x))} \leq \widehat{E}\widetilde{v},$$

where $\widehat{E}$ is the expectation operator taken with the respect to the probability density function $m(v)$. Observe finally that this last inequality holds iff $COV(\widetilde{v}, T'(u(\frac{\widetilde{v}}{n} - x))) \leq 0$, namely iff $T'(u(\frac{v}{n} - x))$ is decreasing in v. This is always true under T concave.■

We have just shown that more risk aversion induces less efforts at the equilibrium. We next examine the effect of more risk on equilibrium efforts. It turns out that an additional condition is required in this case to sign the comparative statics analysis.

Proposition 5 *Consider a risky contest* Γ *in which a unique symmetric equilibrium exists. Then, risk averse agents exert less efforts when the rent is more risky iff* $(\frac{v}{n} - \frac{n}{n-1}\frac{\phi(x)}{\phi'(x)})\frac{u'''(\frac{v}{n}-x)}{-u''(\frac{v}{n}-x)} \leq 2$.

Proof: We need to show that $f(x) = E((\Delta(x)\widetilde{v} - 1)u'(\frac{\widetilde{v}}{n} - x))$ decreases when $\widetilde{v}$ becomes more risky. As is usual in the comparative statics of increasing risk (Rothschild and Stiglitz 1970), this is equivalent to showing that $g(v) = (\Delta(x)v - 1)u'(\frac{v}{n} - x)$ is concave in v. We have $g'(v) = \Delta(x)u'(\frac{v}{n} - x) + \frac{(\Delta(x)v-1)}{n}u''(\frac{v}{n} - x)$ and $g''(v) = 2\frac{\Delta(x)}{n}u''(\frac{v}{n} - x) + \frac{(\Delta(x)v-1)}{n^2}u'''(\frac{v}{n} - x)$. Therefore $g(v)$ is concave iff $\frac{(\Delta(x)v-1)}{\Delta(x)n}\frac{u'''(\frac{v}{n}-x)}{-u''(\frac{v}{n}-x)} \leq 2$ which, by using $\Delta(x) = \frac{n-1}{n^2}\frac{\phi'(x)}{\phi(x)}$ yields the result.■

The condition in the above Proposition places no restriction on the probability distributions for the rent but combines restrictions on the functional forms of u and of ϕ. Yet, this condition holds for all ϕ when $u''' = 0$.[11] However, this last condition on the utility function is unusual in the literature. We now derive another simple sufficient condition based $u''' \geq 0$. This common condition is usually coined "prudence" (Kimball 1990) in the literature,[12] and ensures that the result always holds for all ϕ concave.

Proposition 6 *Consider a risky contest* Γ *in which a unique symmetric equilibrium exists. If* $-w\frac{u'''(w)}{u''(w)} \leq 2$ *, then risk averse and prudent agents always exert less efforts when the rent is more risky under* ϕ *concave.*

Proof: From the concavity of ϕ, we know that $\frac{\phi(x)}{\phi'(x)} \geq x$. This implies that $\frac{v}{n} - \frac{n}{n-1}\frac{\phi(x)}{\phi'(x)} \leq \frac{v}{n} - x$, and thus $(\frac{v}{n} - \frac{n}{n-1}\frac{\phi(x)}{\phi'(x)})\frac{u'''(\frac{v}{n}-x)}{-u''(\frac{v}{n}-x)} \leq (\frac{v}{n} - x)\frac{u'''(\frac{v}{n}-x)}{-u''(\frac{v}{n}-x)} \leq 2$ under $u''' \geq 0$. The result obtained in Proposition 5 concludes the proof.■

Given a constant relative risk aversion (CRRA) utility function (which has $u''' \geq 0$), i.e. $u(w) = (1-\gamma)^{-1}w^{1-\gamma}$ with $\gamma > 0$, the condition $-w\frac{u'''(w)}{u''(w)} \leq 2$ simplifies to $\gamma < 1$.[13] It may sound surprising that the condition assumes that agents are not "too" risk averse to make sure they decrease their efforts when the rent becomes more risky. This result is however not very surprising given previous results in single agent risky settings. Observe indeed that our contest model assumes a multiplicative risk as in a single agent portfolio

[11] This is simply because the left hand side of the inequality condition in Proposition 3 is equal to 0 under $u''' = 0$. Equivalently, it is easy to see from (6) that, when the utility function is quadratic (implying $u''' = 0$), the right hand side always decreases with the variance of the risk.

[12] Sometimes this condition is also called downside risk aversion (Menezes, Geiss and Tressler 1980). Note that DARA implies prudence.

[13] Note that a CRRA utility function displays DARA so that the conditions for uniqueness are satisfied (see Proposition 2).

model. Yet, it has been shown that the comparative statics of the effect of more risk on asset returns in a portfolio model requires a similar condition as the one in the previous Proposition (Rothschild and Stiglitz 1971).

5 Extension to Convex Impact Functions

In this section, we extend the analysis of risky contests to convex impact functions $\phi'' \geq 0$. It is well known in the literature on contests that convex impact functions introduce complications (see, e.g., Cornes and Hartley 2005). In risky contests of the type studied in this paper, the problem with ϕ convex is that the expected payoff function $U_i(x_i, x^*) = Eu(\frac{\phi(x_i)}{\phi(x_i)+(n-1)\phi(x^*)}\widetilde{v} - x_i)$, where x^* is a local maximum, may no longer be concave in x_i. Indeed, as one can see from (4), $U_i(x_i, x^*)$ is concave in x_i if $\phi''(x_i)(\phi(x_i) + (n-1)\phi(x^*)) - 2(\phi'(x_i))^2 \leq 0$ or, equivalently,

$$\frac{\phi(x_i)}{\phi(x_i) + (n-1)\phi(x^*)} \geq \frac{\phi(x_i)\phi''(x_i)}{2(\phi'(x_i))^2}, \tag{7}$$

which may not hold if ϕ is "too" convex.[14] Thus, x^* may not be a global maximum. There are different ways to address this difficulty.[15] One way is to require that agents exert a specific minimum effort level x_0.[16] Before examining the effect of more risk under convex impact functions, we now derive an expression for x_0 in the context of risky games with ϕ convex. This constitutes an independent contribution of this section.

We first establish that: (i) a symmetric solution x^* to the n agents' first order conditions in (3) always exists if the impact function is not "too" convex (Lemma 1) and (ii) x^* is always unique under nonincreasing absolute risk aversion (Lemma 2). To do so, we simply examine the properties of the function f defined in (5).

Lemma 1 *If* $\frac{\phi(x)\phi''(x)}{(\phi'(x))^2} \leq \frac{2}{n}$*, then* $f(x)$ *always has a zero in* $[0, \infty)$*.*

[14] Note that for symmetric efforts, i.e. $x_1 = x_2 = ... = x_n = x$, this condition becomes $\frac{\phi(x)\phi''(x)}{(\phi'(x))^2} \leq \frac{2}{n}$.

[15] The last inequality is necessary and sufficient for U_i concave when the agents are risk neutral. However, U_i can be concave under ϕ convex when agents are "sufficiently" risk averse.

[16] For a graphical illustration of the impact of x_0 in our model, see the appendix.

Proof: First, note that, if $\frac{\phi(x)\phi''(x)}{(\phi'(x))^2} \leq \frac{2}{n}$, the inequality $(\frac{\phi(x)}{\phi'(x)})' = 1 - \frac{\phi(x)\phi''(x)}{(\phi'(x))^2} \geq \frac{n-2}{n}$ implies that $\frac{\phi(x)}{\phi'(x)} > \frac{n-2}{2}x$. For $n > 2$, $\lim_{x\to+\infty} \frac{\phi(x)}{\phi'(x)} = +\infty$ and $\lim_{x\to+\infty} \frac{\phi'(x)}{\phi(x)} = 0$. The proof in Proposition 1 that $f(x) = E((\Delta(x)\widetilde{v} - 1)u'(\frac{\widetilde{v}}{n} - x))$ has at least one zero in $[0, \infty)$ remains valid. [17]■

Lemma 2 *If* $\frac{\phi(x)\phi''(x)}{(\phi'(x))^2} \leq \frac{2}{n}$, *then the zero of* $f(x)$ *is always unique under* u *CARA or* u *DARA.*

Proof: Observe that $\Delta(x) = \frac{n-1}{n^2}\frac{\phi'(x)}{\phi(x)}$ is still a decreasing function since:

$$\Delta'(x) = \frac{n-1}{n^2}\frac{\phi(x)\phi''(x) - (\phi'(x))^2}{(\phi(x))^2} \leq \frac{n-1}{n^2}(\phi'(x))^2\frac{\frac{2}{n} - 1}{(\phi(x))^2} \leq 0.$$

Therefore, the proof in Proposition 2 that $E((\Delta(x)\widetilde{v} - 1)u'(\frac{\widetilde{v}}{n} - x)) = 0$ implies $-E((\Delta(x)\widetilde{v} - 1)u''(\frac{\widetilde{v}}{n} - x)) \leq 0$ under u CARA or u DARA remains valid.■

Now, we prove that with a well defined level of minimum expenditure requirement, the "equilibrium" expected payoff function $U_i(x_i, x^*)$ is concave in x_i on the relevant set of feasible efforts.

Lemma 3 *Let* x^* *be the unique zero of* $f(x)$ *and let* x_0 *be the implicit solution of* $\frac{\phi(x_0)}{\phi(x_0)+(n-1)\phi(x^*)} = M$, *where* $M = \sup(\frac{\phi(x)\phi''(x)}{2(\phi'(x))^2})$. *If* $\frac{\phi(x)\phi''(x)}{(\phi'(x))^2} \leq \frac{2}{n}$, *then* $U_i(x_i, x^*)$ *is a concave function in* x_i *for all* $x_i \in [x_0, \infty)$.

Proof: Let $P(x_i)$ be the increasing function defined as $P(x_i) = \frac{\phi(x_i)}{\phi(x_i)+(n-1)\phi(x^*)}$. Then, for any $x_i \geq x_0$, we have $\frac{\phi(x_i)}{\phi(x_i)+(n-1)\phi(x^*)} \geq \frac{\phi(x_0)}{\phi(x_0)+(n-1)\phi(x^*)} = M$ which implies (7) by definition of M. Finally, $\frac{\phi(x_0)}{\phi(x_0)+(n-1)\phi(x^*)} = P(x_0) = M \leq \frac{1}{n} = P(x^*)$ since $\frac{\phi(x)\phi''(x)}{2(\phi'(x))^2} \leq \frac{2}{n}$, which implies $0 < x_0 \leq x^*$.■

These lemmas directly prove the following result. Note that in the following Proposition the set of feasible efforts is now $X_i = [x_0, \infty)$. As a result,

[17] For $n = 2$, $\ln(\phi)$ is an increasing and concave function. So we have:

$$0 \leq \ln' \phi(x) \leq \ln \phi(x) - \ln \phi(x-1) = \ln \frac{\phi(x)}{\phi(x-1)}.$$

For $\phi(x-1)$ and $\phi(x)$ asymptotically equivalent when $x \to +\infty$, this inequality implies again $\lim_{x\to+\infty} \frac{\phi'(x)}{\phi(x)} = 0$.

the game $\Gamma = \{X_i, U_i, i = 1, 2, ..., n\}$ is thus called the "restricted" risky contest.[18]

Proposition 7 *Let x_0 be the implicit solution of $\frac{\phi(x_0)}{\phi(x_0)+(n-1)\phi(x^*)} = M$, where $M = \sup(\frac{\phi(x)\phi''(x)}{2(\phi'(x))^2})$. If $\frac{\phi(x)\phi''(x)}{(\phi'(x))^2} \leq \frac{2}{n}$, then there always exists an interior symmetric equilibrium in the "restricted" risky contest Γ with $X_i = [x_0, \infty)$. Moreover, the interior symmetric equilibrium is unique under u CARA or u DARA.*

The above result shows that, given a full set of model parameters $\{u, \widetilde{v}, \phi, n\}$, it is always possible to determine a minimal effort level x_0 supporting an interior symmetric equilibrium. Note that the results rely on the assumption $\frac{\phi(x)\phi''(x)}{(\phi'(x))^2} \leq \frac{2}{n}$. Under the standard CSF given by $\phi(x) = x^m$, this is equivalent to $m \leq \frac{n}{n-2}$.

We now move on to the comparative statics analysis and assume that a unique symmetric equilibrium always exists in the risky contest with ϕ convex. We finally show that, contrary to intuition, greater risk may induce risk averse agents to exert greater equilibrium efforts.

Proposition 8 *Consider a risky contest Γ in which a unique symmetric equilibrium exists. If $-w\frac{u'''(w)}{u''(w)} \geq 2$, then risk averse and prudent agents always exert more efforts when the rent becomes more risky under ϕ convex together with $\frac{\phi(x)\phi''(x)}{(\phi'(x))^2} \geq \frac{1}{n}$.*

Proof: First, note that $\frac{\phi(x)}{\phi'(x)} \leq \frac{n-1}{n}x$ since $(\frac{\phi(x)}{\phi'(x)} - \frac{n-1}{n}x)' < 0$ under ϕ convex and $\frac{\phi(x)\phi"(x)}{(\phi'(x))^2} \geq \frac{1}{n}$. This implies that $\frac{v}{n} - \frac{n}{n-1}\frac{\phi(x)}{\phi'(x)} \geq \frac{v}{n} - x$, and thus that $(\frac{v}{n} - \frac{n}{n-1}\frac{\phi(x)}{\phi'(x)})\frac{u'''(\frac{v}{n}-x)}{-u''(\frac{v}{n}-x)} \geq (\frac{v}{n} - x)\frac{u'''(\frac{v}{n}-x)}{-u''(\frac{v}{n}-x)} \geq 2$ under $u''' \geq 0$.■

Note that this result relies on additional conditions on convex ϕ and concave u. Given a CRRA utility function, i.e. $u(w) = (1-\gamma)^{-1}w^{1-\gamma}$ with $\gamma > 0$, the condition $-w\frac{u'''(w)}{u''(w)} > 2$ simplifies to $\gamma > 1$. Moreover, under $\phi(x) = x^m$, the condition $\frac{\phi(x)\phi''(x)}{(\phi'(x))^2} \geq \frac{1}{n}$ simplifies to $m \geq \frac{n}{n-1}$. In other words, this result assumes that the agents are "sufficiently" risk averse, and that the impact functions are "sufficiently" convex. Note that the last condition on the impact functions is not inconsistent with those above ensuring the existence of

[18] Minimum expenditure requirements in rent seeking contests have been studied by Hillman and Samet (1987), Yang (1993), Schoonbeeck and Kooreman (1997), Dari-Mattiacci et al. (2007), and Münster (2007).

a unique equilibrium. However, the combination of these conditions suggests that rent seeking efforts can increase with more risk only under quite special cases.

6 Conclusion

Our paper studies a strategic contest with a risky rent. It delivers a simple message: Risk averse agents usually exert less efforts when they become (more) risk averse and when the rent becomes (more) risky. Importantly, we consider a model with a divisible rent, and assume that agents compete for a share of this risky rent. Hence, our model is different from a probabilistic contest game in which the effect of risk aversion has typically been studied in the literature (e.g., Cornes and Hartley 2012). Moreover, we have examined more complicated cases with convex impact functions (Corchón 2007). In these cases, the message above must be qualified, or even reversed. More precisely, it is possible that risk averse agents exert more efforts when the rent becomes more risky. Finally, we should mention that an important limitation of our analysis is that we have only considered a symmetric game.

7 References

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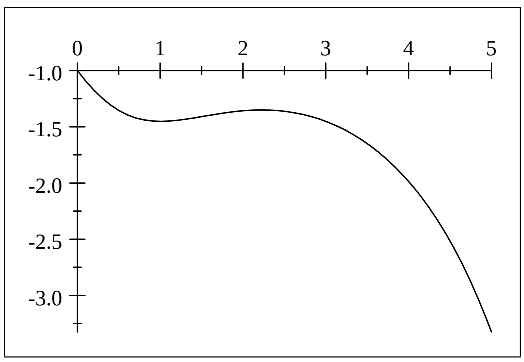
8 Appendix

Figure 1 below illustrates the role of a minimum expenditure requirement in our analysis of risky contests with $n = 3$ agents and ϕ convex. It shows $U_i(x_i, x^*)$ when (i) agents' preferences are represented by a CARA utility function with the form $u(w) = -exp(-rw)$, where $r = 1$, (ii) the values of the risky rent $\widetilde{v}$ are uniformly distributed over the interval $[4, 8]$, and (iii) the impact function is convex with the form $\phi(x) = x^m$, where $m = 1.8$. Here, $x^* = 2.2273$ (computations and figure created using Scientific WorkPlace 5.5).



This figure clearly shows that $x_i = x^*$ is not the best response to $x_{-i} = x^*$ if the set of feasible efforts is $X_i = [0, \infty)$. Indeed, agent i is better off choosing a zero effort level. Thus, x^* is not an equilibrium of the risky contest. However, it is also apparent from this figure that $x_i = x^*$ is the best response to $x_{-i} = x^*$ if a minimum expenditure is required. In other words, $x^* = 2.2273$ might be an equilibrium of the "restricted" risky contest $\Gamma = \{X_i, U_i, i = 1, ..., n\}$ with $X_i = [x_0, \infty)$ and x_0 high enough. For example, $x_0 = 1$ would be suitable.