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A Note on Stein's Overreaction Puzzle

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ABSTRACT

Recently, Christoffersen et al. (2013) argue that the overreaction puzzle of Stein (1989) can be explained by a variance-dependent pricing kernel. In this note, we challenge this view. Our theoretical results are in line with their argument that the variance under risk-neutral measure is more persistent than the variance under physical measure due to a negative variance risk premium. But our results do not support their argument that the more persistent variance is able to qualitatively explain Stein's findings. We show theoretically that the persistence of the volatility cannot amplify the movements of long-term variance to short-term fluctuations in variance, and, therefore, conclude that Stein's overreaction puzzle is still unsolved.

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Motivation

In an influential paper, Stein (1989) derives and empirically tests a model of the term-structure of implied volatility. Assuming a mean-reverting volatility process, under rational expectation, the long-term volatility should move in responsive but smoothing manner to changes in short-term volatility. However, the findings of Stein suggest that for index options, the empirical values of this elasticity exceed the theoretical upper bound of normal reaction. Stein interprets his finding as overreaction; an option market anomaly vis-à-vis rational expectation. Poteshman (2001) extends the term structure model of Stein (1989) and investigates the response of option market investors to new information. He finds that these investors exhibit short-horizon underreaction, long-horizon overreaction and increasing misreaction to information contained in daily changes in the instantaneous variance. More recently, Christoffersen et al. (2013) and Lehnert et al. (2016) replicate the term-structure tests of Stein with more recent data and confirm his earlier results. However, Christoffersen et al. (2013) argue that the finding of Stein does not signal an option anomaly, but is consistent with rational behavior. They use a variance-dependent pricing kernel and generate a negative variance risk premium that is related to both the risk-aversion to equity risk and the risk-aversion to variance risk. They argue that the negative premium for variance is able to qualitatively explain Stein's findings. In this note, we challenge this view. Our theoretical results are in line with their argument that the variance under risk-neutral measure is more persistent than the variance under physical measure due to a negative variance risk premium. But our results do not support their argument that the more persistent variance under risk-neutral measure is able to qualitatively explain Stein's overreaction puzzle. We show theoretically that the persistence of the volatility process cannot amplify the movements of long-term variance to short-term fluctuations in variance, and, therefore, conclude that Stein's overreaction puzzle is still unsolved.

The continuous –time stochastic variance process

In line with Christoffersen et al. (2013), we consider the following dynamics for the spot price $S(t)$

$$dS(t) = \big(r + \mu v(t)\big)S(t)dt + \sqrt{v(t)}S(t)dZ_1(t) \quad (1)$$

$$dv(t) = \kappa\big(\theta - v(t)\big)dt + \sigma\sqrt{v(t)}(\rho dZ_1(t) + \sqrt{1-\rho^2}dZ_2(t)) \quad (2)$$

where $v(t)$ is the instantaneous variance; r is the risk-free rate; μ relates to the equity risk premium; $Z_1(t)$ and $Z_2(t)$ are independent Wiener processes. The variance is assumed to revert towards a long run mean, θ, with a mean reverting speed κ. In a risk-neutral world, the variance also follows a mean reverting process but under the risk-neutral measure

$$dS(t) = rS(t)dt + \sqrt{v(t)}S(t)dZ_1^*(t) \quad (3)$$

$$dv(t) = (\kappa\big(\theta - v(t)\big) - \lambda v(t))dt + \sigma\sqrt{v(t)}(\rho dZ_1^*(t) + \sqrt{1-\rho^2}dZ_2^*(t)) \quad (4)$$

where $Z_1^*(t)$ and $Z_2^*(t)$ are independent Wiener processes under the risk-neutral measure. The unspecified term λ is the variance risk premium.

The proposed pricing kernel in Christoffersen et al. (2013), which is a unique arbitrage-free specification consistent with both the physical and risk-neutral dynamics takes the form

$$M(t) = M(0)\left(\frac{S(t)}{S(0)}\right)^{\phi} \exp\left(\delta t + \eta\int_0^t v(s)ds + \xi\big(v(t) - v(0)\big)\right) \quad (5)$$

where δ and η determine investor's time preference; ϕ and ξ determine the equity risk-aversion and the variance risk-aversion respectively. The variance risk premium, λ, in terms of the risk-aversion parameters ϕ and ξ is derived as

$$\lambda = -\rho\sigma\phi - \sigma^2\xi \quad (6)$$

The equity risk-aversion parameter is negative, $\phi < 0$, since the marginal utility is a decreasing in the spot price; the variance risk-aversion parameter is positive, $\xi > 0$, since the hedging needs increase in times of uncertainty; the correlation between equity return and its variance is empirically found to be negative, $\rho < 0$. Both terms in equation (6), the $\rho\sigma\phi$ and the $\sigma^2\xi$, are positive and thus the variance risk premium is negative.

The variance under risk-neutral measure in equation (4) do not distinguish whether the variance risk premium originates from variance risk-aversion ξ or from equity risk-aversion ϕ. Christoffersen et al. (2013) construct the unique pricing kernel with both risk-aversions and count on the variance risk-aversion to capture the overreaction puzzle. If the variance is constant, their pricing kernel amounts to the standard power utility setup in Rubinstein (1976) and Brennan (1979) with only risk-aversion ϕ and with zero variance risk premium λ. In comparison with the variance-dependent pricing kernel, a standard pricing kernel with only risk-aversion ϕ in a power utility setup that corresponds to the Heston's (1993) stochastic variance dynamics would be

$$M(t) = M(0)\left(\frac{S(t)}{S(0)}\right)^{\phi} \exp\left(\delta t + \eta \int_0^t v(s)ds\right) \tag{7}$$

And the variance risk premium, $\tilde{\lambda}$, amounts to

$$\tilde{\lambda} = -\rho\sigma\phi \tag{8}$$

Christoffersen et al. (2013) note that the variance risk-aversion $\xi > 0$ is not needed for the variance risk premium $\lambda < 0$, as we can also see from either equation (6) or (8). However variance risk-aversion $\xi > 0$, their variance-dependent pricing kernel is a quadratic function of the equity market return, which is then able to capture option market anomalies, including the overreaction puzzle. To be specific, they argue that it is important that the magnitude of the negative variance risk premium, $|\lambda|$, grows as volatility rises ($\xi > 0$), as can be seen from equation (6). The growing negative variance risk premium increases the risk-neutral persistence, as can be seen from equation (4), and therefore amplifies the movements of long-term options values in response to short-term fluctuations in volatility. Christoffersen et al. (2013) thus conjecture that the Stein finding of overreaction does not signal an anomaly but is consistent with their variance-dependent pricing kernel.

We agree with Christoffersen et al. (2013) that the magnitude of the negative variance risk premium grows as volatility rises. We also support their argument that the growing negative variance risk premium increases the persistence of variance under risk-neutral measure. To show it more explicitly in the continuous-time setup, we rewrite the variance dynamics in equation (4) as

$$dv(t) = \kappa^*\big(\theta^* - v(t)\big)dt + \sigma\sqrt{v(t)}(\rho dZ_1^*(t) + \sqrt{1-\rho^2}dZ_2^*(t)) \qquad (9)$$

where $\kappa^* \equiv \kappa + \lambda$ and $\theta^* \equiv \kappa\theta/(\kappa + \lambda)$. Under the risk-neutral pricing probability, the variance reverts towards a long run mean, θ^*, with a mean reverting speed κ^*. A negative variance risk premium leads to a less mean-reverting, as $\kappa^* = \kappa + \lambda$ and, therefore, $\kappa^* = \kappa - \rho\sigma\phi - (\sigma^2\xi\ ,\ 0) < \kappa$.

Next we prove that the persistence of the volatility cannot amplify the movements of long-term variance to short-term fluctuations in variance. The elasticity parameter that determines the variance term structure specified by Stein (1989) remains to be a constant and does not change with the persistence of the volatility. Stein's overreaction phenomenon remains to be a puzzle in this framework, in particular under the pricing kernel proposed by Christoffersen et al. (2013).

The variance term structure

From equation (2), we have the stochastic variance dynamics as

$$dv(t) = \kappa\big(\theta - v(t)\big)dt + \sigma\sqrt{v(t)}(\rho dZ_1(t) + \sqrt{1-\rho^2}dZ_2(t))$$

Using Ito's lemma, the expectation of instantaneous variance as of time $t+\tau$ at time t is

$$E_t\big(v(t+\tau)\big) = e^{-\kappa\tau}(v(t)-\theta)+\theta \tag{10}$$

The conditional variance, V_t^{τ}, is equal to the averaged integrated variance over the time interval $(t, t+\tau)$ as

$$V_t^{\tau} = \frac{1}{\tau}E_t\left[\int_{i=0}^{i=\tau} v(t+i)di\right] = \frac{1-e^{-\kappa\tau}}{\kappa\tau}(v(t)-\theta)+\theta \tag{11}$$

Using the long-run mean and mean reverting speed under risk-neutral pricing probability, i.e. $\kappa^* \equiv \kappa+\lambda$ and $\theta^* \equiv \kappa\theta/(\kappa+\lambda)$, the conditional variance in risk-neutral measure, ${V^*}_t^{\tau}$, is similarly

$${V^*}_t^{\tau} \equiv \frac{1-e^{-(\kappa+\lambda)\tau}}{(\kappa+\lambda)\tau}\left(v(t)-\frac{\kappa\theta}{\kappa+\lambda}\right)+\frac{\kappa\theta}{\kappa+\lambda} \tag{12}$$

Assuming the short-term variance is over time interval $(t, t+\tau_1)$ and the long-term variance over time interval $(t, t+\tau_2)$, the conditional variances are respectively

$${V^*}_t^{\tau_1} = \frac{1-e^{-(\kappa+\lambda)\tau_1}}{(\kappa+\lambda)\tau_1}\left(v(t)-\frac{\kappa\theta}{\kappa+\lambda}\right)+\frac{\kappa\theta}{\kappa+\lambda} \tag{13}$$

$${V^*}_t^{\tau_2} = \frac{1-e^{-(\kappa+\lambda)\tau_2}}{(\kappa+\lambda)\tau_2}\left(v(t)-\frac{\kappa\theta}{\kappa+\lambda}\right)+\frac{\kappa\theta}{\kappa+\lambda} \tag{14}$$

To generalize the equation (11), we include a time difference, j, and view the conditional variance at time t over the time interval $(t+j,\ t+j+\tau)$

$$V_{t+j}^{\tau} = E_t\left[\frac{1}{\tau}E_{t+j}\left[\int_{i=0}^{i=\tau} v(t+j+i)di\right]\right] = \frac{1-e^{-\kappa\tau}}{\kappa\tau}\Big(E_t\big(v(t+j)\big)-\theta\Big)+\theta \tag{15}$$

From equation (10), we have $E_t\big(v(t+\tau)\big) = e^{-\kappa\tau}(v(t)-\theta)+\theta$. Plugging it into equation (15) we get the generic physical variance term structure (VTS)

$$VTS_t \equiv V_{t+j}^{\tau} = \frac{1-e^{-\kappa\tau_1}}{\kappa\tau_1}e^{-\kappa j}(v(t)-\theta)+\theta \tag{16}$$

Using the long-run mean and mean reverting speed under risk-neutral pricing probability, i.e. $\kappa^* \equiv \kappa + \lambda$ and $\theta^* \equiv \kappa\theta/(\kappa + \lambda)$, the generic risk-neutral variance term structure (risk-neutral VTS) is similarly given by

$$VTS_t^* \equiv V^{*\tau_1}_{t+j} = \frac{1-e^{-(\kappa+\lambda)\tau_1}}{(\kappa+\lambda)\tau_1} e^{-(\kappa+\lambda)j}\left(v(t) - \frac{\kappa\theta}{\kappa+\lambda}\right) + \frac{\kappa\theta}{\kappa+\lambda} \tag{17}$$

Equations (17)-(13) lead to

$$V^{*\tau_1}_{t+j} - V^{*\tau_1}_{t} = \frac{1-e^{-(\kappa+\lambda)\tau_1}}{(\kappa+\lambda)\tau_1}(e^{-(\kappa+\lambda)j} - 1)(v(t) - \frac{\kappa\theta}{\kappa+\lambda}) \tag{18}$$

Equations (14)-(13) give

$$V^{*\tau_2}_{t} - V^{*\tau_1}_{t} = (\frac{1-e^{-(\kappa+\lambda)\tau_2}}{(\kappa+\lambda)\tau_2} - \frac{1-e^{-(\kappa+\lambda)\tau_1}}{(\kappa+\lambda)\tau_1})(v(t) - \frac{\kappa\theta}{\kappa+\lambda}) \tag{19}$$

Rearranging (18) and (19) we have

$$V^{*\tau_1}_{t+j} - V^{*\tau_1}_{t} = \frac{\tau_2(1-e^{-(\kappa+\lambda)\tau_1})(e^{-(\kappa+\lambda)j}-1)}{\tau_1(1-e^{-(\kappa+\lambda)\tau_2}) - \tau_2(1-e^{-(\kappa+\lambda)\tau_1})}(V^{*\tau_2}_{t} - V^{*\tau_1}_{t}) \tag{20}$$

At this step, we can plug in the variance risk premium derived from Christoffersen et al.'s (2013) variance-dependent pricing kernel, namely $\lambda = -\rho\sigma\phi - \sigma^2\xi$. We write the variance term-structure that is testable in the same way as shown in Stein (1989) as

$$E_t\left[\left(V^{*\tau_1}_{t+j} - V^{*\tau_1}_{t}\right) - \beta(V^{*\tau_2}_{t} - V^{*\tau_1}_{t})\right] = 0$$

where the elasticity parameter is defined as $\beta \equiv \frac{\tau_2(1-e^{(\rho\sigma\phi+\sigma^2\xi-\kappa)\tau_1})(e^{(\rho\sigma\phi+\sigma^2\xi-\kappa)j}-1)}{\tau_1(1-e^{(\rho\sigma\phi+\sigma^2\xi-\kappa)\tau_2}) - \tau_2(1-e^{(\rho\sigma\phi+\sigma^2\xi-\kappa)\tau_1})}$. The elasticity parameter is a non-linear function of equity risk-aversion, variance risk-aversion, correlation between equity return shock and variance shock, volatility of the variance, volatility time horizons, and physical variance mean-reverting speed.

We illustrate the theoretical values of elasticity parameter in several scenarios in Table I. We show that the variance risk-aversion $\xi > 0$ from the pricing kernel by Christoffersen et al. (2013) increases the persistence of the variance much more, in comparison with that from a standard pricing kernel with only the equity risk-aversion ϕ, as we can see from the examples given by Column 2 v.s. 3, or by Column 4 v.s. 5 in Table I. However the different degrees of persistence do not necessarily impose a difference on the value of the elasticity parameter. We show that the coefficient is a constant, $\beta = 2$, for the scenario that the short-term variance horizon is 1-month and long-term is 2-month as used by Stein (1989). Therefore, we get exactly the same testable equation as in Stein (1989) as follows:

$$E_t\left[\left({V^*}_{t+4w}^{1m} - {V^*}_t^{1m}\right) - 2({V^*}_t^{2m} - {V^*}_t^{1m})\right] = 0$$

The changing of mean-reverting speed and risk-aversion coefficients under risk-neutral probability in comparison with those under physical probability, thus the magnitude of the persistence of the variance, does not change the value of the elasticity parameter in this scenario. We can have a more persistent variance under the risk-neutral measure compared with that under physical measure, as $\kappa^* = \kappa + \lambda\kappa^* = \kappa - \rho\sigma\phi - (\sigma^2\xi\ ,\ 0) < \kappa$. But we cannot produce the Stein finding of overreaction through the elasticity parameter β being equal to two. Therefore, it is not the variance risk-aversion ξ, or the negative variance risk premium λ, or the magnitude of the persistence of the variance process that can produce the Stein finding of overreaction.

Conclusion

Recently, it was argued that the overreaction puzzle of Stein (1989) can be explained by a variance-dependent pricing kernel. In this note, we challenge this view. Christoffersen et al. (2013) use a variance-dependent pricing kernel and generate a negative variance risk premium that is related to both the risk-aversion to equity risk and the risk-aversion to variance risk. They argued that the negative premium for variance is able to qualitatively explain Stein's overreaction puzzle and that the stylized fact does not signal an anomaly but is consistent with the model they developed with the variance-dependent pricing kernel. Our theoretical results are in line with their argument that the variance under risk-neutral measure is more persistent than the variance under physical measure due to a negative variance risk premium. But we do not support their argument that the more persistent variance is able to qualitatively explain Stein's overreaction puzzle. The model of Christoffersen et al. (2013) would be able to explain the puzzle only if their variance-dependent pricing kernel can help to generate an elasticity parameter larger than two for the term structure. But this is impossible no matter how strongly the variance risk premium affects the persistence of the variance in the model. Therefore, we claim that the model by Christoffersen et al (2013) cannot produce the Stein finding of overreaction through the theoretical predictions in their framework. We conclude that Stein's overreaction puzzle is still unsolved.

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Table I

Values of elasticity parameter $\beta \equiv \frac{\tau_2(1-e^{(\rho\sigma\phi+\sigma^2\xi-\kappa)\tau_1})(e^{(\rho\sigma\phi+\sigma^2\xi-\kappa)j}-1)}{\tau_1(1-e^{(\rho\sigma\phi+\sigma^2\xi-\kappa)\tau_2})-\tau_2(1-e^{(\rho\sigma\phi+\sigma^2\xi-\kappa)\tau_1})}$

where τ_1 is the short-term variance horizon; τ_2 is the long-term variance horizon; j is the time difference in weeks; ρ is correlation between equity return shock and variance shock; σ is volatility of the variance; ϕ is equity risk-aversion; ξ is variance risk-aversion; κ is the mean-reverting speed of the variance time series on a weekly basis; the $persistence \equiv e^{(\rho\sigma\phi+\sigma^2\xi-\kappa)}$ is the persistence of the variance time series on a weekly basis. For illustration, τ_1 is 1 month and τ_2 is 2-month, the same horizons used in Stein (1989) and in Christoffersen et al (2013).

j = No. of Weeks	$\xi = 0$	$\sigma^2\xi = 0.05$	$\xi = 0$	$\sigma^2\xi = 0.05$
	$\rho\sigma\phi - \kappa = -0.25$ $\rho\sigma\phi + \sigma^2\xi - \kappa = -0.20$		$\rho\sigma\phi - \kappa = -0.1$ $\rho\sigma\phi + \sigma^2\xi - \kappa = -0.05$	
	$persistence = 0.78$	$persistence = 0.82$	$persistence = 0.90$	$persistence = 0.95$
1	0.699	0.658	0.577	0.538
2	1.245	1.197	1.099	1.049
3	1.669	1.639	1.572	1.537
4	2	2	2	2